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EXPERIMENTAL STUDY OF TORSIONAL COLUMN FAILURE

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SUMMARY

Thirty-three 24ST aluminum-alloy 2- by 2- by 0.10-inch channels, with lengths ranging from 10 to 90 inches were tested at Stanford University in compression to obtain an experimental verification of the theoretical formulas for torsional failure developed by Eugene E. Lundquist of the N.A.C.A. The observed critical loads and twist-axis locations were sufficiently close to the values obtained from the formulas to establish the substantial validity of the latter. The differences between observed and computed results were small enough to be accounted for by small and mostly unavoidable differences between actual test conditions and those assumed in deriving the formulas. Some data were obtained from the shorter specimens regarding the growth of the buckles that resulted in local buckling failure.

INTRODUCTION

Two desirable features for any structural member are that it should be easily connected to other members and that all portions of its surface should be conveniently accessible for inspection and the application of protective coatings. Both objectives can be much more easily attained by the use of open sections, such as channels, angles, and I-beams, than with closed sections, such as tubes and boxes. Unfortunately for the engineer who wishes to use open sections, many of them when tested in compression, have shown a tendency to fail by twisting under much lower loads than those indicated by the formulas covering the better understood types of column failure. This result has produced a well-grounded prejudice against the use of open column sections, in general, since no one can predict with confidence how they will act under load. The situation calls for an experimentally validated theory of torsional failure that would permit the designer to devise open sections which would be better for his purposes than any closed ones, whenever that is possible.

Until quite recently little progress had been made in developing the theory of torsional failure. One of the best early attempts was that of Leduc reported in reference 1. This research was followed by the more important work of H. Wagner and his associates (references 2 and 3) who have developed methods of computing the critical load with respect to torsional failure and have done some experimental work to validate their formulas. The Wagner formula is based on the assumption that, in torsional failure, the center of rotation of each cross section will be at the shear center of the section. Lundquist and Fligg (references 4 and 5) point out that the axis of rotation will take such a location as causes the critical load to be a minimum and that Wagner's equation for the critical stress could be used in this more general case if certain terms are redefined. Kappus (reference 6) has also tackled the problem of torsional instability and obtained the same results as Lundquist and Fligg. Kappus, however, gives a much more extended mathematical treatment of the problem than given in any other publication.

Up to the present time, nearly all of the work done on the problem of torsional failure in this country has been of a theoretical character. Shortly after the publication of reference 1, Mr. James G. Sutherland and Mr. Warren G. Clark tested as flat-end columns a few angle and "hat" sections that they believed would fail torsionally. The hat sections tested by Clark showed little if any tendency to fail in that manner. The angle sections tested by Sutherland did fail torsionally and he obtained some interesting data showing the character of the deformations of the specimens under load. He was, however, unable to check Leduc's formulas or that of Pugsley in reference 7. The results of the work of Sutherland and Clark were embodied in theses submitted to Stanford University in partial fulfillment of the requirements for the degree of Engineer. They have received no further publication on account of the inability of the authors to check existing theory and defects in their methods of test, which introduced some uncertainty regarding the proper interpretation of the results. Some experimental work on torsional failure has also been done at the University of Michigan by A. Zahorski (reference 8), who tested semicircular columns with flat ends.

In the summer of 1937, the N.A.C.A. asked the writer to undertake an experimental check of the Wagner equation for torsional instability as generalized by Lundquist and his associates. The present report covers this experimental work.

The writer wishes to acknowledge assistance which he has received from others and which has contributed materially to the successful outcome of the tests. Special mention is due to Mr. E. E. Lundquist for suggestions regarding the design of the end fittings, theoretical data regarding the action of the specimens, and various suggestions pertaining to the best manner of obtaining the desired results. Thanks are also given to Professor H. A. Williams of Stanford University for assistance in devising and constructing special test jigs and in preparing the report; to the Larsson Machine Tool Co. of Berkeley, Calif., for suggestions regarding the detailed design of the end fittings and the careful and accurate construction of those fittings; to Professor M. S. Hugo of Stanford University for help in the detailed design of the end fittings and checking the accuracy of their construction; to Professor A. B. Domonoske for help in the detailed design of the end fittings; to Dr. L. B. Tuckerman of the National Bureau of Standards for advice regarding the design of the knife edges; to Mr. R. L. Templin of the Aluminum Company of America for suggestions regarding the preparation of the test specimens; to Mr. O. G. Warm for design and construction of various parts of the testing and measuring apparatus; and to Messrs. R. O. Brittan, J. S. Dunning, M. Miner, W. G. Vincenti, and R. J. Wellman for intelligent and conscientious labor in helping to carry out the tests and in working up the test data.

APPARATUS AND TEST PROCEDURE

Specimens

Since the major objective of the investigation was to check theoretical formulas for critical load and location of axis of twist for centrally loaded columns, the primary tests were directed toward the determination of those quantities. Secondary tests were carried out to check the quality of the material and to determine the torsion constant of the section used.

Thirty-three column tests were made on 11 different lengths of 24ST aluminum-alloy extruded channels, three specimens boing tested in each length. Each specimen was identified by a number consisting of its length in inches followed by a hyphen and the numeral 1, 2, or 3. All 33 specimens were cut from six 20-foot channels, their lengths

varying from 10 to 90 inches. The nominal "mid-line" dimensions of the cross section used were: width of back and width of flange, 2.00 inches each; thickness of back and flanges, 0.100 inch. The check of the cross-sectional dimensions described in the appendix showed that, although the section was not absolutely uniform, the variation was small and the nominal dimensions formed the most satisfactory basis for computations of geometrical section properties.

The quality of material was determined from test coupons cut from apparently uninjured portions of the column test specimens. Three of these coupons were obtained from each of the six original lengths of material for use in tension tests. An additional coupon was used to determine the shearing modulus by a torsion test.

Column-Test Apparatus

The column tests were carried out in the 200,000-pound Riehle testing machine at Stanford University. The general arrangement of the apparatus used is shown in figure 1, which is a photograph of a 90-inch specimen under load. Load was applied to the specimen A from the moving head of the testing machine through the upper end fitting B. From the specimen, the load passed through the lower end fitting C to the 20,000-pound capacity hydraulic capsule D, and its magnitude was indicated by the Bourdon tube gage E. The hydraulic capsule and the Bourdon gage were mounted on a pair of 8-inch steel channels clamped to the weighing table of the testing machine. The hydraulic capsule and Bourdon tube gage are standard articles supplied by the A. H. Emery Co. They were used instead of the lever system of the testing machine for measuring load because of their greater precision. Rotation and translational movement of selected cross sections of the specimen were determined from measurements of the distances from points on the antennas F attached to the specimen, to reference points on the wood scaffolding G,G, clamped to the standards of the testing machine.

End fittings.- The end fittings used to obtain the desired boundary conditions were specially designed and constructed for the tests according to suggestions of Mr. E. E. Lundquist of the N.A.C.A. The main requirements were that the resultant load should be applied through the centroids of the end cross sections, and that the end cross

sections should be free to warp, only the midpoints of each of the three main elements being constrained to remain in a plane.

Figure 2 shows the two end fitting assemblies, with the exception of the bearing blocks that were in direct contact with the ends of the specimen. The action of the fitting can be visualized best from the left-hand assembly from which the locking plates have been removed. The three main subassemblies are the base A, the gimbal ring B, and the saddle C. The lower base rests directly on the top of the capsule and is held in position by two small bolts. The upper base hangs from the movable head, being held in place by a 5/8-inch bolt through a hole in its center. From the base A, the load is transferred through knife edges into the gimbal ring B. Entering the gimbal ring at its sides, it passes out through its ends and into the saddle C through knife edges. The knife edges of both the base and the gimbal-ring assemblies are rectangular section bars of Bethlehem tool steel of Rockwell hardness C-61, 0.265 inch on a side and 2 inches long. The corresponding seats are cylindrical grooves ground in rectangular plates of the same material. The thickness of these plates at the base of the grooves is approximately three-sixteenth inch. The positions of knife edges and seats are so located that all knife edges are within 0.0005 inch of the plane of the end of the specimen, and the intersection of the knife-edge lines forms a right angle with its apex at the centroidal axis of the specimen.

In order to facilitate handling the end fittings and setting up the specimens for test, two locking plates D, are attached to the gimbal ring by screws and washers, the washers being used to provide clearance between the plates and the upright standards of the base and thus avoid friction that would prevent unrestrained rotation about the knife edges when the locking screws had been removed. The locking screws pass through these plates to holes in the vertical standards of the base, thus providing a positive method of connecting the gimbal ring to the base. Similar but smaller plates E are used to connect the saddle to the gimbal ring. These plates and locking screws are shown in position on the right-hand fitting of figure 2. The locking plates and screws did not prevent all play between the members but were fully effective in preventing the upper fitting from falling apart when no specimen was in the testing machine and greatly facilitated placing a specimen in the proper position. During a test when the locking

screws were removed, the parts could rotate freely about the knife edges.

It being feared that a specimen might collapse when the locking screws were not in place, or through some other accident the upper fitting might fall apart and be damaged, screws F and G were inserted through the large holes in the locking plates D and E of that fitting and into threaded holes tapped into the base and the gimbal ring. A clearance of about three-sixteenth inch prevented these screws from carrying any load unless there was failure of some part of the fitting. Fortunately, they were never called into play in this manner.

Figure 3 shows a saddle removed from the gimbal ring. The load passes from the knife-edge seats at the ends to the "inner knife-edge assembly" at the center. This assembly carries two knife edges. The longer one passes under the midpoints of the flanges of the specimen and the shorter one under the midpoint of the back. The photograph also shows the inner knife-edge assembly from the other end fitting by itself. The inner knife edges are similar to the ones supported by the base and the gimbal ring except that they are shorter and the cross section is only 0.177 inch instead of 0.265 inch on a side. The lengths of these knife edges were such as to provide a working length of about 1 inch for each of the three elements of the cross section of the specimen. Like the other knife edges, their working edges are within 0.0005 inch of the plane of the end of the specimen. The end fittings were carefully constructed so that the inner knife edge in the plane of symmetry of the specimen was in line with the knife edges supporting the saddle. The entire inner knife-edge assembly, however, could be moved through a range of about one-eighth inch by loosening one screw and tightening another. This movement, which was parallel to the plane of symmetry, was provided on account of uncertainty regarding the exact location of the centroid of the specimen and the consequent desirability of being able to adjust the position of the resultant load with respect to the specimen.

From the inner knife edges, the load was transmitted to the specimen through three of the bearing blocks shown in figure 4. The upper group of blocks in the photograph show how they nested together when placed on the inner knife edges. The three other blocks show the shape of these units, which were interchangeable. Each block has a deep groove with sloping sides and flat bottom in the upper

portion to receive the end of the back or one flange of a specimen. In the lower portion is a shallower groove with parallel sides and rounded top at right angles to the first. The flat bottom of the upper groove is just wide enough to permit bearing of the end of the specimen. In fact, it was necessary to break the edges of the specimen with a file to get good bearing. The rounded top of the lower groove served as a seat for one of the inner knife edges. The top of the lower and the bottom of the upper groove were in practically the same plane, though small holes at the intersections indicate slight differences in their elevations. The sides of these blocks were beveled to permit at least $\pm 5^{\circ}$ of rotation about the supporting knife edges without mutual interference, thus permitting the desired free warping of the end cross sections. The blocks were made of Nitralloy G with a scleroscope hardness of 92.

Other column-test apparatus.- The antennas used for measuring the rotation of the specimen under load were constructed from round steel rods. Figure 5 shows one antenna assembled on a short section of channel and another disassembled. In order to attach an antenna, it was necessary to drill a 3/16-inch hole in the center of the back of the specimen, and this hole may have had a slight effect on the test results. The area affected is so small that any such effect is believed to be negligible.

The movements of the antennas were determined by measuring the distances from reference points about one-half inch from the end of each arm to fixed reference points on the wooden scaffolding. The reference points on the antennas were marked by scratches on the rods, those on the scaffolding, by ordinary carpet tacks with shallow drilled holes in their heads. The distances between the two were measured by ordinary vernier calipers with special lozenge-shaped attachments on their jaws. One of these calipers is shown assembled and the other disassembled in figure 6.

The over-all distance across the free edges of the flanges was measured by the special calipers shown in figure 6, built up around a 0.001-inch Ames dial. This instrument was called the "M gage" and in this report the readings taken with it are termed, for brevity, the "M readings."

The change in length of the specimen under load was measured by Ames dials attached to the end fittings and registering the movement of those parts with respect to the

scaffolding. (These dials are later shown in place in figures 17 and 18.)

Longitudinal strains were measured near the center of most of the specimens by Huggenberger tensiometers.

Column-Test Procedure

After the individual specimens had been cut from the six original channels, the ends were squared on a milling machine and the edges of the end cross sections broken with a file so they would fit in the grooves of the bearing blocks. Equally spaced holes for the antennas were then drilled through the center line of the back. For specimens 30 inches or more in length, five antennas were used; for lengths from 16 to 24 inches, three antennas were used; and for the 10-inch specimens, a single antenna was provided. The reduction in number of antennas for the shorter lengths was due to the crowding that would have resulted had five antennas been installed.

The remaining steps in the preparation of a specimen for test included marking positions for the tensiometers and M readings and taking and recording a set of M readings under zero load. In general, the M readings were taken near the centers of the segments delimited by the antennas. The tensiometers were located near the middle antenna.

After the specimen had been placed in the testing machine and sufficient load had been applied to take up all play, the locking screws were removed from the end fittings and the end fittings were checked to make sure that the parts were in their proper relative positions. Usually some adjustment of the end fittings was found necessary, but this adjustment could be easily made as long as the axial load did not exceed 300 or 400 pounds. These adjustments were made by eye, because it was found that, if the load were too well centered, the rotations would be so small that the precision in locating the center of rotation would be undesirably poor.

Once the specimen was properly located in the testing machine, the antennas and the tensiometers were attached and, if necessary, pins were inserted between the Ames dials for measuring change in specimen length and their reference points on the scaffolding. The load was then increased, usually by running the moving head of the testing machine

at the rate of 0.1 inch per minute, until somewhat more than half of the expected critical load had been reached. The gimbal rings and the saddles of the end fittings were tapped sharply at this point to help get the specimen fully seated in the end fittings. The load was then reduced to a convenient "basic load" for starting the tests. At the same time, the end fittings were rechecked to make sure that the various parts were in the proper relative positions.

With the specimen subjected to the basic load, the first step was to set to zero the dials for measuring change in length. The upper of these dials was called the J and the lower, the K dial; their readings were called the J and K readings. At the same time, the tensiometers were set at 1.50, the highest convenient point on the scale. The stage was thus set for taking the readings with the vernier calipers. Four readings, distinguished by the letters A, B, C, and D, were taken for each antenna. The position of the caliper for each of these readings is shown diagrammatically in figure 7.

In taking the vernier readings, the observer placed one end of a lozenge-shaped attachment in the hole drilled in the reference tack and held it there firmly while setting the movable jaw of the caliper for the reading. To make this setting, he swung the caliper in a small arc while moving the jaw with the slow motion screw until the point of the other lozenge-shaped attachment barely scratched against the antenna arm at the proper reference mark. Difficulty in obtaining accurate readings by this method was anticipated and a small indicator was attached to one caliper jaw in place of the lozenge-shaped attachment. This method proved unsatisfactory, however, as the spring of the indicator, though apparently very flexible, was too stiff; and the observer would hear or feel the contact of the caliper with the antenna several thousandths of an inch before the indicator would register contact. At first, the observers found it difficult to check their vernier measurements and some time was devoted to practice before the reported tests were commenced. Since the reliability of the vernier readings continued to increase with practice, the quality of the data obtained improved as the test program was carried out. Even in the earliest tests, however, checks were applied to the readings as they were being taken and doubtful readings were repeated until those checks were satisfied.

The first of these checks consisted in taking all vernier readings under the basic load twice, repeating readings at any point where the two values differed by more than 0.002 inch. In later tests on the shorter members, the basic readings were often repeated when the first two values differed by only 0.001 or 0.002 inch. This precaution was not a very time-consuming process because, in the later tests, the two readings at more than half the reading stations were identical. The second check, which was applied at loads other than the basic, was an application of the fact that by taking four readings on each antenna, two independent measures of its rotation were obtained. If, when the angle of rotation was small, these differed by more than about 0.007 radian (about 2.5′ of arc) the readings were repeated. When the rotation was large, the permissible error was increased.

In the first few tests the practice was to take a set of readings for level I (i.e., the readings for the top antenna), following with those for levels II, III, IV, and V in succession, and then take the check readings in the same order. When this procedure was followed, the checks between the first and the second readings were often not so good as was desired, and it was noted that usually the load indicated by the gage had changed from 10 to 50 pounds during the period between the two sets of readings. Although part of the errors may have been due to the inexperience of the observers, most of it was considered to be due to an actual change of load and a corresponding actual change in the deformation of the specimen.

Various phenomena indicated that this change in load was due to temperature changes and a resulting unequal thermal expansion of the steel screws of the testing machine and the aluminum-alloy specimen. For example, when the temperature dropped, as when the specimen had to be left in the testing machine over night, the load would drop; but, in the mornings or through the lunch period while the temperature was rising, the load would increase. The only difficulty with this theory is that it would indicate that the resulting changes in load would be as great for the shorter as for the longer specimens, but this result was not the case. This difficulty may well have been due to the testing of the shorter specimens in more equable weather when there was less change in temperature and to the fact that the tests were made in much shorter periods of time because the crew became more experienced in the work and the number of vernier readings

was decreased with the reduction in the number of antennas used. The difficulty was surmounted by taking both the original and the check readings at level I before taking any readings at level II and so on until all the readings for the basic load had been taken. This method gave less time for temperature effects to develop and consequently the number of additional readings required was much reduced.

The vernier readings were followed by a group of M readings made with the special calipers designed for the purpose and check readings of the tensiometers and the J and the K dials.

After the basic readings were completed, the load was increased by lowering the moving head of the testing machine at the rate of 0.05 inch per minute. At convenient intervals, the moving head would be stopped and a set of readings taken. At each stop, the normal procedure was to tap the gimbal rings and saddles with a wooden mallet, read the J and K dials and tensiometers, take the vernier readings, take the M readings, and then check the J and the K dials and the tensiometers. At first, this procedure took nearly half an hour but, by the time the last specimens with five antennas were tested, it took only about 10 or 12 minutes.

As the vernier readings at a given level were being taken, the observer tapped the indicating needle of the load gage with the maximum reading needle and read the load, which was recorded with the vernier readings. Sometimes there was an appreciable change in load while a complete set of readings for the five levels was being taken, but there would be little change in any one level. In the earlier tests, when the change in load amounted to more than 20 or 30 pounds, it was attempted to restore the original load by raising or lowering the moving head. On account of the difficulty of restoring the original load accurately, readings for the different levels were obtained for somewhat different though not greatly divergent loads without changing the position of the moving head.

In the earlier stages of a test, when the rotations were not apparent to the eye, the moving head would be stopped near predetermined values of load, the increments depending on the length of specimen. As the load and the rotation increased, the increments were determined by holding a foot rule so that it would be touched by the B arm

of one of the antennas after the rotation had increased a predetermined amount. The amount of rotation allowed between readings varied, being small at first and increasing as the load and the rotation increased. The object was to locate the points on the experimental curves so as to get the proper shapes of those curves as well as possible.

With the longer specimens, a time came when a large increase of rotation would be produced with very little increase in load and often with an accompanying decrease. This situation was accepted as representing failure and the movement of the testing machine would be reversed until the column had been relieved of most of its load. At least one set of readings would then be taken at a load close to the basic load to determine roughly the amount of permanent set. In some tests, two such sets of readings were taken, one under a load a little greater and the other under a load a little less than the basic load. The load was then entirely removed, the specimen taken out of the testing machine, and a final set of M readings taken.

In the first test, the tensiometers were shaken off the specimen when it failed and, in the following tests, they were removed before their readings indicated the likelihood of failure. With a few of the longer specimens, the tensiometers were left on until after the maximum load had been determined and the final readings at a load close to the basic load had been taken.

With the columns of 24-inch and shorter lengths, there was relatively little rotation and the procedure was varied as follows: Basic-load readings were taken at about 1,000 pounds. At about 2,000 pounds, the moving head was stopped and only dials J and K and the tensiometers were read. At about 3,000 pounds a complete set of readings was taken. The load was then increased to about 8,000 or 10,000 pounds, stopping sometimes to take only the J and K and tensiometer readings and sometimes to take a complete set. In the neighborhood of 8,000 or 10,000 pounds, the tensiometers would be removed, because they would have served their purpose of showing that there was no excessive eccentricity of loading and there was danger of their being injured if the specimen buckled. Also, by this time local buckles of the free edges of the flanges would begin to be visible and it was considered more important to have a more complete set of M readings than to get more data from the tensiometers. From this point on the buckles would be closely watched as the moving head was lowered, and some-

times it would be stopped to take only the J, K, and M readings and sometimes to take the vernier readings also. Complete vernier readings were omitted under the lower loads because the rotations were so small that the readings would be of little help in determining the axis of rotation and also be of little value for computing the critical load by the Lundquist extension of Southwell's method (reference 9). The complete readings under about 3,000 pounds were to provide a kind of secondary set of basic readings.

At some load, one of the buckles in the free edges of a flange would suddenly increase and the specimen would start to collapse. A complete set of readings would be taken at this point, including an M reading at the widest part of the buckle taken with an ordinary steel scale. The moving head would then be raised to reduce the load on the specimen. In some tests, a set of readings was taken when the resistance had been reduced to approximately the basic load but, in the later tests, these were omitted as being of no special value. The specimen was then taken out of the machine and a final set of M readings was taken and recorded.

The procedure just described was normally followed in the tests. Various deviations from this procedure were made in specific tests, usually to obtain some special information. These deviations will be described in connection with the discussions of the special data obtained from such tests.

The first specimen tested was 22-2, and the data from this test are the least reliable of all. The moving head was then raised and the scaffolding modified to take the 90-inch specimens. After the 90-inch group, the other groups were tested in order of decreasing length until the three 10-inch specimens had been tested. In each group, normal practice was to test the three specimens in numerical order. After test 24-1 was completed, the saddles were removed from the testing machine to permit an adjustment to be made in the position of the inner knife edges. In practically all tests made up to that time, what bending in the plane of symmetry had been noted was in nearly every case away from the axis of twist. The saddles were replaced after moving the inner knife edges a small amount in the direction needed to reduce the eccentricity of loading. The tensiometer and the deflection readings in the following tests indicated that the desired result had been accomplished.

Torsion Tests

The method of making the torsion tests is shown in figure 8. A steel bar A was screwed to the outer surface of the back at each end of the specimen. Near the outer end of one of these bars, 10 inches from the plane of symmetry of the channel, a load was transmitted to the bar through a knife edge from the hanger B. Vertical reactions were applied at the lower ends of the screws C, which passed through the steel bars in the plane of symmetry of the specimen. The necessary downward force required for equilibrium was supplied by a cord from the outer end of the other bar A tied to a weight resting on the floor. In order to minimize friction the supporting screws C rested on steel blocks. The amount of twist was measured over a 15-inch length in the middle of the specimen by the movement of the pointer D along the scale E. This scale was graduated in radians and was placed so that its center would be collinear with the ends of the supporting screws C. The small spirit level F was attached to the loading arm so that the arm could be brought to a horizontal position before taking each reading.

As the loading arms A overhung the ends of the specimen, it was possible to reverse the positions of the supporting screws C and make tests with the center of rotation at the centroidal axis of the specimen.

The torsion test of a flat specimen was made with the same apparatus, but in that case the load hanger B, was moved to a point 2.00 inches from the center line of the specimen.

TEST RESULTS

The main objectives of the column tests were to determine the critical loads and the positions of the axes of twist. The critical loads were read directly from the dial of the hydraulic weighing system. They are listed in table I and shown graphically in figure 9. The locations of the axes of twist were determined from the vernier readings by the method outlined later; they are also listed in table I and are plotted in figure 10.

Method of Determining Location of Twist Axis

Differences between successive vernier readings were actually measures of the changes in distance from the reference points on the scaffolding to the corresponding reference points on the antennas. They were also assumed to be measures of the movements of the reference points on the antennas perpendicular to the positions assumed by the antennas' arms when the specimen was under the basic load. As long as the translational movements were small compared with the actual vernier readings, and the rotations of the antennas were also small, the error resulting from this assumption was negligible. Throughout most of the tests, both of these conditions existed. The only times when the assumption introduced appreciable error was when, under the critical load, the longer specimens rotated through relatively large angles with practically no change in axial load. No correction was made in such cases because considerable extra computation would have been necessary, and it was sufficient to know that the rotation was large and changing rapidly with load without having precise quantitative information on the point.

Figure 11 shows the "trunk" of an antenna in its positions under two successive loads. The distances A and B represent the changes in vernier readings which measured the movements of the antenna reference points a and b. Point O is a point midway between a and b, and e is the position of the centroid of the cross section of the specimen, 0.76 inch from O. For convenience, the distances A and B are shown greatly exaggerated in comparison with the distance between reference points a and b. Also both A and B are shown as positive, i.e., implying that both antenna readings increased, although in the tests whenever one of these readings increased the other usually decreased. The angle of rotation, θ is evidently equal to 0.05 (A-B). The distance y_+, the movement of a point d on the antenna trunk at a distance T from O, is found by simple geometry to be

$$y_+ = 0.50\ (A + B) + 0.05\ T\ (A - B)$$

The movement of a point d near the specimen is needed to locate the axis of twist conveniently. For the earlier tests, T was computed so that d was on the theoretical axis of twist. It was soon decided, however, that nothing was to be gained by this procedure and, for the later tests,

the computation was made for the point for which T equaled 4.00 inches. The movement of this point is then given by the relation

$$y_+ = 0.50\ (A + B) + 0.20\ (A - B)$$

Similarly, the movement of e at the centroid of the cross section where T = 0.76 inch is given by

$$y_c = 0.50\ (A + B) + 0.038\ (A - B)$$

The vernier readings to the C and D reference points on the "cross arms" of the antennas were used to check the rotation θ and to determine the translational movement x of the cross section parallel to the axis of symmetry. The C readings were taken from points 7-3/8 inches, and the D readings from points 7-1/8 inches from the plane of symmetry. The resulting formulas were therefore

$$\theta = (D - C)/14.5$$

and

$$x = 0.50\ (C + D) + (D - C)/116$$

For the two measures of rotation to agree it is necessary that $D - C = 0.725\ (A - B)$, and this relationship was used to check the accuracy of the vernier readings during the course of the tests. The amount by which this check was not satisfied was termed Δ and was a rough measure of the reliability of the group of readings from which it was computed.

The positions of the twist axes were determined graphically from the computed values of y_+ and y_c. A base line was first laid off to represent the distance between the points for which these values were determined. The distances y_+ were laid off on a perpendicular at one end of this base line and the distances y_c were laid off on a perpendicular at the other end, both to a convenient scale that exaggerated the rotation of the antenna. The lines connecting the corresponding plotted values of y_+ and y_c constituted a sheaf of vectors representing the positions of the antenna trunk under the various loads. For the longer specimens, most of these lines passed through or very close to a point that could be accepted as representing the position of the twist axis. A representative vec-

tor sheaf is illustrated by figure 12. For the shorter specimens, the results were not so reliable, owing to the small amount of rotation and the resulting greater influence of small errors in making the vernier readings. Consequently, it was generally more difficult and sometimes impossible to select a satisfactory point as the observed location of the axis of twist. The twist axis locations listed in table I are the distances in inches from the centroid as determined by applying this method to the middle antenna vernier readings. For each specimen, a column indicates qualitatively the precision of the tabulated value.

The twist-axis locations indicated by the vector sheaves for the other antennas differed little from the tabulated values. The existing differences tended to show that, as the ends of the specimen were approached, the distance to the twist axis was slightly reduced. For the most part, the differences were less than the probable errors in determining the distance in question, and no quantitative conclusions can be developed from them.

Action of Specimens Under Axial Load

Types of failure.- Two distinct types of failure were encountered in the compression tests. The longer columns failed torsionally and the shorter ones by local buckling of one flange. With the longer specimens, under low loads the rotations could be detected only from the changes in the vernier readings and the shortening of the specimen was almost directly proportional to the load applied. As the critical load was approached, it became possible to see the antennas rotate as the moving head of the testing machine was lowered. At the same time, the increment of load resistance developed by a given increment of shortening continuously decreased until it was possible to obtain a large increase in both twist and shortening with no measurable increase in resistance developed. In some cases the continued lowering of the moving head resulted in a small decrease in the resistance developed, but this action took place only when the midsection of the specimen had twisted through a fairly large angle, in most cases 10° or more. Except that the movement of the cross section was primarily one of rotation rather than of translation, the action was very similar to that of the center of a long closed-section column as the Euler load is approached. In figure 13 are three representative $P - \theta$ curves showing

graphically the relation between the axial load P in pounds and the rotation of the midsection θ in degrees and radians. When the rotation of the midsection became very large, 25° or so, the ends of the channel flanges began to bear on the sides of the bearing-block grooves. The relative motion of the bearing blocks also became so large that mutual interference developed. Both of these factors caused such changes in the end conditions that no attempt was made to continue the tests until the specimens collapsed.

The action of the longer specimens under load is illustrated in figure 1 and in figures 14 to 18. Figure 1 shows specimen 90-3 under maximum load. The amount of twist is clearly indicated by the ends of the antennas, which were on a straight line when the basic load was applied. Figure 14 was taken at the same time as figure 1, but from the opposite side of the specimen. Figure 15 shows specimen 70-1 subjected to 2,940 pounds axial load before the tensiometers were removed. After the tensiometers had been taken off, the moving head was lowered, causing additional twist but no additional resistance; the photographs of figure 16 were then taken. Nearly all of the load was then removed and the specimen reverted to practically its original shape, as shown by figure 17. Figure 18 shows photographs of a 50-inch specimen under the maximum load. Figure 18(a) is a front view that shows the amount of twist. Figure 18(b) is a side view showing that the deflection in the plane of symmetry accompanying this twist was negligible.

After the specimen had twisted a certain amount, the internal forces were expected to be so distributed that one flange would be subjected to excessive compression and would collapse by local buckling. No such action took place in any test in which much twisting occurred. In the test of specimen 30-3, the downward motion of the moving head was continued for some time after the maximum load had been reached. This motion caused a large amount of rotation and waves began to form in the free edges of the flanges. One of these waves was comparable in depth with those associated with the local buckling failures of the shorter specimens. In the test in question, however, the drop off in load was negligible even though the moving head was lowered until the rotation of the middle cross section exceeded 20°. Most of the waves in the flanges disappeared as the moving head was raised in removing the load. The largest wave, however, remained and can be seen in the longest of the specimens shown in figure 19.

The specimens less than 30 inches in length failed by local buckling of one flange. As the load was applied, some rotation was indicated by the vernier readings but its magnitude was normally less than that shown by the longer specimens below the knee of the $P - \theta$ curve. As the load approached the critical, waves began to develop in the free edges of the flanges, usually becoming definitely recognizable to the naked eye more than 1,000 pounds before the critical load was reached. For lengths between 16 and 24 inches, the normal condition was that each flange developed at least two complete waves, whereas only one developed in the 10-inch lengths. In general, these waves were symmetrical, both flanges buckling in or both flanges buckling out at any given distance from the end of the specimen. As the load increased, the amplitude of these waves increased at an accelerating rate until the critical load was reached. The approach of the critical load was also foreshadowed by a considerable increase in the observed rotations of the antennas, though they remained small in comparison with those exhibited by the columns for which the failure was primarily torsional. At the critical load, one flange failed suddenly as the result of a large increase in the size of one of the buckles. In some tests the failure took place as the result of an increase in the amplitude of the wave, which appeared deepest just before the critical load was reached. In many of the tests, however, the local buckle that produced failure came at an unexpected location. With several of the specimens, the inward buckles were much more pronounced up to the point of failure than were the outward buckles but all of them failed by buckling outward, as can be seen from figure 19. The flange in which failure took place was evidently determined by the direction of rotation of the cross section, since the buckle invariably appeared in the flange on the side toward which the rotation was directed. This result is illustrated by figure 20, which shows the buckling failure of a 10-inch specimen.

Data recorded.- In addition to the maximum load carried and the observed location of the twist axis, in table I are listed the following additional data pertinent to the axial-load tests of both long and short specimens: maximum deflection, parallel to the plane of symmetry, of the middle cross section under loads not exceeding 90 percent of the maximum; direction of rotation; maximum change in M readings under increasing load prior to buckling; and type of failure. When the near end of an antenna appeared to move to the right of an observer, the rotation was consid-

ered positive and is represented by a plus sign; rotation in the opposite direction is indicated by the minus sign. All specimens failed either torsionally or by local buckling, the two types of failure being indicated by the letters T and B.

In table II are listed data that apply only to the specimens that failed by local buckling and to specimen 30-3, in which the test load also produced a permanent buckle in one flange. In this table are recorded the first load at which definite buckling of the flanges was noticed in the tests, the load carried by the specimen immediately after buckling, M readings at the widest part of the buckle taken under that load and after removing the specimen from the testing apparatus, and the flange in which the buckle appeared.

Copies of complete log sheets of the tests including the individual vernier, tensiometer, J and K dial, and M gage readings, and the tabulated computations of θ, y_+, y_c, and Δ are on file at the Guggenheim Aeronautic Laboratory at Stanford University.

Supplementary Column Tests

With one or two of the longer specimens, after the reported test was completed, the load was increased and the column carried practically as much load as in the first run. After specimen 70-1 had been subjected to the regular test, the locking pins were inserted in the end fittings and the column was reloaded. For this supplementary test, therefore, the ends were "encastré but free to warp." In the supplementary test, the critical load was 4,500 pounds, which exceeded the critical for the standard test by 1,570 pounds. The maximum rotation in this test exceeded 40°. The same procedure was carried out with specimen 40-1 and a load of 6,700 pounds was developed in the supplementary test, an increase of 900 pounds over that developed in the original test. This time the maximum angle of twist was not measured, but it was quite large. When specimen 40-2 was treated in the same manner, the maximum load developed was only 5,730 pounds, 70 pounds less than the specimen had carried in the original test and the accompanying rotation of about 14° indicated that the maximum had been reached.

Since specimens 40-1 and 40-2 failed at practically

the same load in the standard test, in the test of specimen 40-3 the locking screws were allowed to remain in the end fittings. They were completely in place while the load was being changed but, as each set of readings was taken, they were checked to see if they could be easily turned by hand. Little binding developed on these screws until a load of 5,810 pounds was reached, indicating that the end cross sections had no appreciable tendency to rotate. Up to this load of 5,810 pounds, the vernier readings showed that the specimen acted in almost exactly the same manner as specimens 40-1 and 40-2. Beyond this load, however, the locking screws came into action and helped restrain the member. As a result, the load continued to increase to 6,700 pounds, when the test was stopped. The maximum load recorded for this test in table I is 5,800 pounds because it appeared reasonable to believe that, if the locking pins had been removed, that would have been the maximum load carried.

Tension Tests of Material

The properties of the material as determined from the tension test are summarized in the following table. The results from the individual test specimens are tabulated in the appendix.

Results of Tension Tests of Material

Property	Young's modulus (lb./sq.in.)	Yield point (lb./sq.in.)	Ultimate stress (lb./sq.in.)	Elongation in 2 in. (percent)
Average	10,200,000	48,400	62,300	14.0
Maximum	11,120,000	51,100	66,700	16.5
Minimum	9,660,000	46,000	58,400	11.5

From this table, it is seen that the values of Young's modulus deviated from 9.2 percent above to 3.5 percent below the average. Yield point varied from 11.2 percent above to 5.0 percent below the average. Ultimate tensile stress varied from 7.1 percent above to 6.3 percent below the average.

Torsion Tests

Three torsion tests were made on a length of the complete channel section. In one of these tests, the supports

were on the centroidal axis, in a second they were 2.50 inches, and in the third they were 3.00 inches below that axis. In all three tests, the rotation was 0.00010 radian per inch of length per inch pound of torque. Defining GJ as TL/θ where T is the torque in inch pounds, L the length of the member, and θ the angle of twist, the tests indicated GJ to equal 10,000.

Since the fillets at the re-entrant angles of the channel section are of very small radius, it was assumed that the shearing modulus of elasticity would be determined from these tests on the assumption that the torsional characteristics of the channel section would be identical with those of a flat section 6.00 inches wide and 0.100 inch thick, and that Timoshenko's formula 64' on page 77 of reference 10 would be applicable. This indicated that the shearing modulus

$$G = \frac{3M\ L}{b\ c^3\ \theta} = \frac{3 \times 50 \times 15}{6 \times 0.001 \times 0.075} = 5,000,000 \text{ lb./sq. in.}$$

Substitution in equation 81 on page 90 of the same reference reduced the computed value of G to 4,650,000 pounds per square inch.

These values for G appeared unreasonably large and a check test was made on a rectangular specimen averaging 1.765 inches in width and 0.100 inch in thickness. This specimen twisted 0.091 radian in a length of 15 inches under a torque of 16 inch-pounds. Substitution in equation 64' of reference 10 gave the value of G as 4,480,000 pounds per square inch. Substitution in equation 81 gave G = 4,450,000 pounds per square inch. For purposes of interpreting the column test results, the observed value of G was therefore assumed to be 4,500,000 pounds per square inch.

PRECISION OF RESULTS

The dial of the Bourdon gage used for measuring load in the column tests is graduated to 50 pounds, but the load could be estimated with reasonable accuracy to the nearest 10 pounds. In many of the earlier tests, it was found that owing to temperature change the load would vary while a set of readings was being taken although the position of the moving head was not modified. The Bourdon gage was rather

sluggish in responding to this change and, although the load could be read to the nearest 10 pounds, it is considered that the readings are reliable only to the nearest 20 pounds.

During the period that elapsed between running tests 16-2 and 16-3, the hydraulic weighing system was calibrated by a representative of the Pelton Water Wheel Co., who used the 20,000-pound capacity Morehouse proving ring No. 91 for the purpose. This calibration showed the indicated loads to be correct within one-half of 1 percent, the indicated load being almost invariably less than the actual load. In the analysis of the test data no adjustments were made to reflect the results of this calibration. The precision of the recorded figures for axial load is therefore within 20 pounds or one-half of 1 percent, whichever value is the greater.

The vernier micrometers were read to the nearest 0.001 inch. At first the observers had some trouble in checking their readings because it was difficult for them to tell exactly when the vernier jaw just touched the antenna arm. The operation was practiced until the observers could consistently check themselves within 0.002 inch before the reported tests were started. As the test program proceeded, the observers became more expert, and the precision of the vernier readings increased. By the time the tests were concluded, most of the readings were accurate to within 0.001 inch. This fact is shown by the results of the last hundred check readings taken under basic loads. Sixty-five of these check readings were identical with the original observations, thirty-one differed from the original observations by 0.001, three by 0.002, and one by 0.003 inch.

The great majority of the vernier readings were undoubtedly correct to within 0.002 inch. Each rotation check was based on the addition or subtraction of eight separate vernier readings. Had all of these readings been 0.002 inch in error in the unfavorable direction, the resulting value of Δ would have been 13.8

If the precision of the vernier readings is taken as ± 0.002 inch, that of the translational movements of the cross section, y_+, y_c, and x may be assumed at the same figure. The resulting percentage errors in these quantities, particularly x, would be very large where these movements are small, the possible error often being larger than the movement being measured. This fact is considered not to be serious since the primary objective of

the tests was to measure rotations rather than translations and, whenever the translational movements were of appreciable magnitude, the percentage errors in their observed values were reasonable in size. If the measurement of translational movements had been a major objective of the tests, other methods would have been employed.

Owing to the length of the antenna trunk between the reference points for the A and B vernier readings, the precision of the rotation measurements was considerably better than that of the translation measurements. Even when Δ was equal to 10.0, which was seldom the case except when the rotation was relatively large, the possible error in the angle of rotation θ was only 14.5/20000 = 0.000725 radian or 2' 29". Even this small amount was often a considerable percentage of the rotation of the longer specimens under low loads or that of the shorter specimens under loads considerably below the critical; but, whenever the rotation was appreciable, the percentage error in its measurement was negligible.

It is practically impossible to determine the precision of the observed locations of the twist axis from the precision of the vernier readings. The only valid information on this point can be obtained from inspection of the vector sheaves. In some of these sheaves, all the vectors apparently pass through a single point. In most cases, however, there was no single point that could be taken as a common intersection, but a reasonably good estimate of the center of rotation could be made; particularly if one or two of the vectors were disregarded. For some specimens, an observed twist-axis location is recorded although its precision is rather poor; whereas, for three of the specimens, the vectors showed no signs of having a common intersection. On the whole, the observed locations that fall close to the plotted curve of predicted values were obtained from the better intersections and they are considered correct within about 0.10 inch.

Practically all of the observed values of Young's modulus, tensile yield point, and ultimate tensile strength are within 10 percent of the average values. Of these quantities, only Young's modulus affects the critical load in torsional instability or the position of the axis of twist, and the effect of a 10-percent error in E on the latter quantity is quite small.

DISCUSSION OF RESULTS

The observed values of critical load and twist-axis location are of little practical value by themselves because they apply to a single size of member tested with very artificial end conditions. The importance of these results lies in the degree to which they confirm the validity of the theoretical formulas for the section and the loading conditions employed. No attempt has been made to correlate the test results with any formulas for torsional failure except those of Lundquist given in the following section.

Theoretical Formulas

For the special case of a channel subjected to the test conditions the axis of rotation should lie in the plane of symmetry at the distance Q from the centroid of the cross section obtainable from the expression

$$X^2 + 2\left[\frac{1}{\pi^2}\,\frac{G}{E}\left(\frac{L}{h}\right)^2 \frac{h}{b}\,\frac{A}{F} + \frac{H}{F}\,\frac{D - B}{D}\,\frac{M}{}\right] X - \frac{B}{D} = 0 \qquad (1)$$

where

$X = Q/h$

E, Young's modulus.

G, shearing modulus.

$$A = \left[\frac{1}{3} + \frac{2}{3}\,\frac{b}{h}\left(\frac{t_b}{t_h}\right)^3\right]\left(\frac{t_h}{h}\right)^2$$

$$B = \frac{1}{3\,D}\left(\frac{b}{h}\right)^3 \left(\frac{t_b}{t_h}\right)\left[\frac{b}{h}\,\frac{t_b}{t_h} + 2\right]$$

$$D = 2\,\frac{b}{h}\,\frac{t_b}{t_h} + 1$$

$$F = -\left[\frac{1}{6}\,\frac{d}{h}\left(6\,\frac{t_b}{t_h} + \frac{h}{b}\right) + \frac{1}{2}\,\frac{b}{h}\,\frac{t_b}{t_h}\right]$$

$$H = \frac{1}{12}\left(\frac{d}{h}\right)^2 \left(6\,\frac{t_b}{t_h} + \frac{h}{b}\right) + \frac{1}{6}\left(\frac{b}{h}\right)^2 \frac{t_b}{t_h} + \frac{1}{2}\,\frac{d}{h}\,\frac{b}{h}\,\frac{t_b}{t_h}$$

$$\frac{d}{h} = \frac{1}{D}\left(\frac{b}{h}\right)^2 \frac{t_b}{t_h}$$

$$M = \frac{1}{12}\left(6\,\frac{t_b}{t_h} + \frac{h}{b}\right)$$

b, h, t_b, and t_h are cross-sectional dimensions as shown in figure 11.

L, length of specimen.

For the 2 by 2 by 0.100 channel section used in the tests, equation (1) reduces to

$$X^2 - 2\left[\frac{1}{\pi^2}\,\frac{G}{E}\left(\frac{L}{h}\right)^2 \frac{9}{3200} + \frac{95}{384}\right] X - \frac{11}{36} = 0 \qquad \text{(1a)}$$

The critical stress at which torsional instability takes place is given by equation (2)

$$f_{cr} = K\,G + K_b\,\frac{E\,\pi^2}{(L/h)^2} \qquad (2)$$

where

$$K = \frac{A}{B + D\,X^2}$$

$$K_b = \frac{M\,X^2 + F\,X + H}{(h/b)\,(B + D\,X^2)}$$

For the section used in the tests the expressions for K and K_b become

$$K = \frac{0.030}{11 + 36X^2}$$

$$K_b = \frac{63X^2 - 96X + 4^3}{99 + 324X^2}$$

Comparison of Computed and Observed Results

The observed and the computed values of critical load and distance Q are shown in table III. The values of P_{cr} in column 2 are the maximum loads experimentally obtained. The computed values designated P_1 were obtained by substituting the standard handbook values, E=10,300,000 and G = 3,800,000 pounds per square inch, in equations (1) and (2). They therefore represent the critical loads that would be predicted from the formulas in the normal process of design. The critical loads P_2 were obtained in the same manner except that they are based on E = 10,200,000 and G = 4,500,000 pounds per square inch, values that were obtained from tests on the material used. In the computation of the critical loads P_3, equation (1) was disregarded, the observed values of Q listed in column 7 being employed to calculate X for use in equation (2). The observed values of E and G rather than the standard values of those quantities were used in the computation of P_3. The values of Q listed in columns 8 and 9 are those computed in calculating the critical loads P_1 and P_2, respectively.

In figure 9 the observed critical loads are plotted as ordinates with the lengths of the specimens as abscissas. Two curves of predicted load are also shown in that figure. The upper curve, labeled P_e, is the Euler column curve for the section tested and indicates the critical loads that would have been expected had the columns failed due to elastic instability in bending without torsion. The lower curve represents the theoretical loads P_2 computed as described. Had a similar curve been drawn to represent the theoretical loads P_1 based on the standard material properties, it would have been a little below the curve of P_2. In some respects the theoretical loads P_3 based on the observed values of Q are the most significant for determining the validity of the formulas from the test results. It would be difficult, however, to draw a satisfactory curve of P_3 since each computed value applies to a specific test and not to all three specimens of a group. Comparisons of the observed critical loads and P_3 must be made from the tabulated values rather than from graphic charts.

In figure 9 different symbols are used to distinguish the specimens that failed torsionally from those that failed

by local buckling. It will be noted that all specimens 30 inches or more in length failed torsionally, while those 24 inches or less in length failed by local buckling. Inspection of table III and figure 9 shows that all of the specimens that failed torsionally except 90-1 and 90-3 did so under loads exceeding those predicted on the basis of the observed properties of the material used. The discrepancy in the case of specimen 90-1 was only 12 pounds, or about one-half of 1 percent. The critical load for this member was appreciably greater than that predicted on the basis of standard material properties. The critical load of specimen 90-3 was greater than the value predicted on the basis of standard material properties but was about 4.4 percent below that predicted on the basis of observed material properties. The other specimens failed at loads exceeding the values predicted on the basis of observed material properties by from 1.7 to 13.7 percent. The excess with respect to the loads predicted on the basis of standard material properties was somewhat greater in every case.

Although the three 24-inch and two of the 22-inch specimens carried more than the predicted loads based on observed material properties, they exhibited little rotation and their failures were definitely of the local buckling type. Two of the 24-inch specimens developed the highest ratios of observed load to predicted load obtained in the series of tests. In general, the shorter the specimen the higher this ratio was found to be, except for the shortest lengths for which the predicted critical loads for torsional failure were well in excess of the loads necessary to produce local buckling.

The specimens of the three shortest groups all failed at loads well below the critical loads for twisting failure. This result was particularly true of the 16-inch and the 10-inch specimens for which the critical loads in torsion were obviously in excess of the loads that would cause local buckling.

Figure 10 shows the observed values of Q and a curve showing the computed values of that quantity based on the observed properties of the material used. It will be noticed that, except in the case of specimen 90-1, the observed values of Q exceed the computed values for all specimens that failed torsionally. If 0.15 inch were added to each of these computed values of Q, they would check the observed values with remarkable closeness, the

agreement being within the precision of the observed values. The normal difference of about 0.15 inch between the computed and the observed values of Q is 7.5 percent of the width of side or back of the specimen.

If, instead of the computed values Q_1 and Q_2, the observed values of Q are used, as was done in computing P_3, the agreement between the predicted and the observed values of critical load is somewhat improved for all specimens that failed torsionally except those of the 90-inch group. The improvement is, however, small because the results of applying equation (2) are little affected by an increase of about 10 percent in the value of Q or X.

The discrepancies between the observed and the computed values of Q for the specimens that failed by local buckling are, in several cases, much greater than those for the specimens that failed torsionally. This result is hardly surprising since twisting was not the primary form of deformation of these specimens and the observed angles of twist were so small that the twist-axis locations were not at all well defined. In fact, the surprising fact is that the observed locations of the twist axes came as close as they did to the theoretical ones; inspection of table I and figure 10 shows that the better the location of the twist axis was defined, the more closely it agreed with its theoretical position.

In the foregoing comparisons of predicted and observed critical loads, the predicted loads have been obtained from the formulas for torsional-instability failure. It is of interest to compare the observed critical loads with those that would be predicted by the familiar Euler formula. From figure 9 it can be seen that, for the columns investigated, the Euler formula indicates critical loads so far above the loads which caused torsional instability as to be entirely inapplicable. On the other hand, the discrepancies between the observed critical loads and the formulas for torsional instability are of such minor magnitude that, in general, these formulas are obviously applicable and are definitely pertinent to the design of structural members of the type under consideration, at least for intermediate lengths. As the length of the specimen increases, the ratio of the observed critical load to the Euler load increases and, for lengths considerably in excess of 90 inches, the failure would probably be by bending with the critical load indicated more accurately by the Euler than by the torsional-instability formula.

Factors Affecting Validity of Predicted Results

When the normal amount of scattering of the plotted points representing the results of a series of column tests is considered, the results of the tests under discussion are gratifyingly convergent. It is true that the observed critical loads are rather consistently in excess of those predicted from the formulas and this result may be due either to some minor errors in the derivation of the formulas, the use of incorrect values for the material properties, minor differences between the boundary conditions assumed in deriving the formulas and those actually provided in the test, or a combination of these factors. The differences between the observed and the computed critical loads are, however, small enough that the tests may be considered to have proved the general validity of the formulas they were intended to check and to encourage designers to use them, and the other formulas based on the same general theory, with considerable confidence.

One possible important source of the discrepancies between the observed and the predicted critical loads was the use of incorrect values for the elastic properties of the material. Although the "observed" values of E and G used in the computations were obtained from tests of coupons cut from the column test specimens, some question exists regarding their validity. The value of Young's modulus E was obtained from tension tests, whereas it would have been better to have used compression tests inasmuch as a difference between the moduli for tension and compression has been found. This difference, however, is not very great and, if E were assumed to be 10,660,000 instead of 10,200,000, the increase in predicted critical loads would be not more than about 3-1/2 percent at most.

The possible error in the observed value of the shearing modulus G is greater than that in Young's modulus E; it may be noted that the observed value of 4,500,000 is 18.4 percent in excess of the standard value of 3,800,000. Both of these values, however, are open to suspicion owing to the lack of development of the technique of testing to determine the shearing modulus. In the past, the customary method of determining G has been by measuring the angle of twist of a round rod or tube. Little or no attention has been paid to the problem of obtaining this quantity from a rectangular section. In the present series of tests, however, the shearing modulus had to be obtained from flat sheets because it would have been imprac-

ticable to have satisfactorily machined round rods from the original channels. Although the tests to determine G were made with care, there is reason to doubt the complete appropriateness of the formulas used to obtain that quantity from the test data and the value 4,500,000 may well be too high. While the use of a lower value of G would reduce the computed critical loads, comparison of the values of P_1 and P_2 in table III will show that the reduction in computed critical load would be much less proportionately than the reduction in G.

It would be interesting, if possible, to make a more thorough study of the elastic properties of the material actually used in the tests to determine more reliable values of both E and G, particularly for G, but such a procedure is hardly necessary to demonstrate the essential validity of the formulas for torsional instability. On the whole, the differences between the predicted and observed critical loads can be adequately explained as the result of unavoidable differences between the assumed and the actual end conditions.

At least two such differences existed that would probably act to increase the experimentally determined critical loads. Most obvious, perhaps, is the existence of friction between the knife edges and their bearings. This friction introduced a certain amount of restraint which was not allowed for in computing the critical loads but which would tend to increase those loads in the same manner that friction in the end fittings tends to increase the critical load of a long slender column that fails by bending. A little light is thrown on this phase of the problem by the results of the tests in which a specimen was loaded while the locking pins remained in place. In most of these tests, the maximum load carried was considerably increased; but the presence of the locking pins appeared to make little if any difference in the load at which torsion became easily visible, their influence seeming to be mainly exerted after the specimen had begun to twist considerably.

Another deviation of the actual from the assumed boundary conditions is that the end fittings were designed to function in a theoretically perfect manner if the back and flanges of the specimen could be considered to have negligible thickness. Thus, the end cross sections were free to warp if it could be thought of as a geometric line. Actually it has finite thickness, the effect of which is to introduce a slight restraint.

Another deviation of the actual boundary conditions from those assumed in the theory, but one more likely to cause a decrease than an increase in the critical load, was that the resultant load was applied at a small distance from the centroidal axis. The presence of such an eccentricity of loading is revealed by the twisting and the deflection parallel to the plane of symmetry that took place before the critical load was reached. Had the determination of the critical loads been the only major objective of the tests, attempts would have been made to eliminate the eccentricity of loading by more careful centering of the specimens in the test apparatus. In this investigation, however, it was considered equally important to determine the location of the center of twist. In the test of specimen 90-2 the centering happened to be nearly perfect and, as can be seen from figure 13, the rotations and the translational deflections of the antennas were hardly measurable until practically the entire critical load had been reached. As a result, the degree of precision obtained in reading the verniers represented relatively large percentage errors in the computed results and, although the data from this test show that the rotations and deflections were negligible, this test was one in which the twist axis could not be located. The data from tests 90-1 and 90-3 in which the centering was not so good proved acceptable for that purpose. It was therefore decided not to attempt to center the specimens with meticulous care but to be satisfied with a centering that would result in measurable rotations throughout most of the loading range and yet not result in excessive translation. In other words, the centering was considered satisfactory if the torsional deformations were obviously of much greater importance than those due to bending. Owing to the careful construction of the end fittings, this result was easily obtained. The fact that it was obtained is shown by the relatively small translations parallel to the plane of symmetry that are listed in table I. In this table it will be seen that up to 90 percent of the critical load none of the specimens that failed torsionally deflected more than 1/1500 of its length.

An interesting characteristic of the translational movements of the antennas was that they often changed in direction when the rotations became large. This result was particularly evident when the translational movement under low loads was away from the center of twist, as it was with nearly all of the longer specimens. The phenomenon was due to the fact that, as the cross section rotated

about a point behind the back of the channel, the centroid moved to the rear a distance equal to that between the centroid and the center of rotation multiplied by the versed sine of the angle of rotation. When the angle of rotation was small, this quantity was less than the translational movement due to bending but, with a large angle of twist, the effect of the twisting overshadowed that of bending.

While the rotations of the specimens indicated the presence of small eccentricities of loading, their directions showed that they did not result from any constant tendency to place the resultant load on one side of the centroidal axis. Of the 33 specimens tested, 15 twisted toward the right of an observer and 18 to the left. In 10 of the 11 lengths tested, 2 of the specimens twisted in one direction and 1 in the other. Only with the 40-inch specimens, was the twist in the same direction in all three tests.

It was recognized that the eccentricity of the loading may have caused measurable differences between the maximum loads actually carried and the loads that would have been carried had the centering been perfect; all the tests were therefore analyzed by a modification of the procedure described by Lundquist in reference 9. This procedure was developed to determine the critical load of a column subject to failure by bending from the translational deflections observed under load. The chief modification was to use the observed rotations in radians in place of observed deflections in inches.. A minor modification was to plot the values of $\Delta\theta$ as ordinates and those of $\Delta\theta/\Delta P$ as abscissas so that the slope of the straight line drawn through the plotted points, instead of the reciprocal of that slope, would represent the difference between the critical load and that at which $\Delta\theta$ and ΔP were taken as zero.

The critical loads obtained by this procedure are listed under the heading P_s in table III and are found to exceed the observed maximum loads by from 1 to 6 percent for the specimens that failed torsionally. The results for the specimens that failed by local buckling were not so consistent. The plotted points diverged more from straight lines and, in one case, the critical load obtained by this method was nearly 10 percent less than that actually carried. There is really no valid reason why the procedure should indicate the critical load with ideal cen-

tering when the failure is by local buckling. The fact that the loads obtained by it differed from the observed maxima by not more than 15 percent in these cases is a defect rather than a merit because it makes it more difficult to determine the true range of applicability of the method.

The three specimens of each group being practically identical, in each group the values of P_s are expected to be closer together than the observed values of P_{cr} that would be affected by the changes in the eccentricity of loading. Of the six groups that failed torsionally, three show less spread between the values of P_s than between those of P_{cr}; whereas, for the other three groups, the reverse is the case. In the five groups that failed by local buckling four showed less spread between the values of P_{cr} than between those of P_s and the difference in the other group was only that between 1,550 and 1,510 pounds. Incidentally these were the largest differences found in the critical load values for any group of three specimens.

Local Buckling Failure

With the shorter specimens that failed by local buckling, the amount of twist was negligible until failure took place and the most valuable information to be obtained from the tests is that pertaining to the buckling. In the first test carried out (22-2), the buckling came as a surprise, no such action having been noted prior to the failure. This test was followed by those of the longer specimens and, when the shorter specimens were again reached, the deformations were being more carefully watched and the growth of the buckles was noted before failure in every case.

In table IV the difference between the load at which the buckles were first definitely noticed and the critical load is recorded. The specific values range from 1,080 to 8,890 pounds. Although the lower of these values applies to one of the 30-inch and the higher to one of the 10-inch specimens, no clearly defined relation indicates when the presence of buckles is to be expected. This feature is not very surprising because it is difficult to devise a criterion for the beginning of the development of a buckle; the load at which the buckle would first become visible would depend largely on the imperfections of the

specimen. As is stated in the appendix, the original specimens showed considerable variations in the distance across the free edges of the flanges and this variation would have an important influence on the nature of the buckling phenomena.

One of the assumptions underlying Lundquist's formulas for torsional failure is that the shape of the cross section remains unchanged. The M readings were taken to determine the degree of validity of this assumption. With the specimens that failed torsionally, the assumption appeared justified as there was very little change in the M readings, at least until the twisting became excessive. Usually the M readings under the basic load differed by a few thousandths of an inch from those taken under zero load, but from then on the change was negligible. The insignificance of these changes can be seen from the values in table I, in which the maximum change in the set of M readings that showed the greatest variation is recorded for each test.

With the shorter specimens, however, the M readings taken near the crest or trough of one of the waves of local buckling exhibited relatively large changes and reflected the growth of the buckles. It would have been desirable to have taken M readings at all such points, but the positions of the waves could not be predicted in advance, and interference with the antennas and tensiometers made it impracticable to take readings at the desired points after the waves had begun to develop. In a few tests, however, notably those of the 10-inch series, the M readings happened to be taken where they showed the growth of the buckling wave very well. The variation of these values in tests 10-1 and 10-2 are shown in figure 21.

When the shorter members failed by local buckling, the resistance to shortening suddenly dropped to about half the critical load. This ratio of load developed after buckling to critical load varied from 40 percent in test 24-2 to 59 percent in test 20-2.

When the buckling failure took place, the distance across the free edges of the flanges at the deepest part of the buckle was measured and was found to vary from 2-35/64 to 2-49/64, the values for the individual tests being listed in table II. After the column had been removed from the testing machine, this distance was again measured with the results listed in table II. These show

that the depth of the bulge decreased by from 8/64 to 18/64 inch but, in most of the tests, the reduction was between 12/64 and 15/64 inch.

One phase of the investigation was a rough determination of the permanent set resulting from the tests. With those of the longer specimens that failed torsionally and were not subjected to large angles of twist, the permanent set was with difficulty, if at all, visible to the naked eye. The longer specimens that were subjected to considerable twist could be seen to have been permanently deformed by sighting along one flange after the test had been completed. In every case, however, the vernier readings taken at approximately the basic load after the critical load had been reached indicated that some permanent set had taken place. Specimen 30-3 developed a large permanent buckle when it was subjected to a large amount of twist under the maximum load, and all the shorter specimens that failed by local buckling showed considerable permanent set after the load had been removed. The amount and character of this permanent set is shown in figure 19, which is a photograph of the shorter specimens taken at the conclusion of the tests.

CONCLUSIONS

1. The tests tend to validate the theoretical formulas developed by Lundquist to cover torsional failure of columns.

2. The discrepancies between the results observed in the tests and those computed from the formulas are not too large to be accounted for by small and mostly unavoidable differences between the conditions of the tests and those assumed in developing the formulas.

3. Designers may use with confidence formulas for torsional failure developed by the procedure employed by Lundquist provided that formulas based on suitable boundary conditions are selected.

4. The importance of torsional failures of open sections is shown by the fact that the critical loads of the specimens that failed torsionally were far below those indicated by the usual column formulas.

Daniel Guggenheim Aeronautical Laboratory,
Stanford University, June 1939.

APPENDIX

PROPERTIES OF THE SPECIMENS

Dimensions

The specimens were cut from six 20-foot lengths of 24ST aluminum-alloy extruded channels specially designed for the tests. The nominal "midline" dimensions of the cross section were: width of back, 2.00 inches; width of flange, 2.00 inches; thickness of back and flanges, 0.100 inch. In order to determine the deviations of the actual specimens from nominal dimensions, measurements with micrometer calipers reading to 0.001 inch were made at cross sections spaced about 6 inches apart, at least three sections being checked on each specimen. The locations of these measurements are shown in figure 22. All of the measurements shown on that figure except E were taken at each section. Measurement E was taken only at the sections near the ends of the specimens, as very little variation was found in that quantity. The results of these measurements are shown in table V, in which are listed the nominal, median, minimum, and maximum values found.

From table V it will be seen that the thickness of the material varied from 0.097 to 0.105 inch. Much of this variation was due to the size of the hole in the die. The thickness at any one measuring point did not vary more than 0.004 inch, while the median values varied from 0.098 to 0.102 inch. These measurements showed that the flanges were thickest near the free edges. From the edges, the thickness decreased for about a third of the distance to the back, at which point it began to increase again. The resulting shape of the section of the flange, greatly exaggerated, is shown in figure 23.

The over-all dimensions of the sections exhibited more variation than the thickness of the material. The over-all widths of flange and back had variations of 0.014 inch. The largest variation was in the over-all width across the free edges of the flanges, which amounted to 0.080 inch. This variation was not at all regular and in the individual specimens ranged from 0.007 to 0.052 inch. This condition indicated a certain amount of waviness in the shape of the free edges of the flanges, which introduced a deviation from ideal conditions that could not be avoided. Table VI is a list of the specimens showing the maximum spread be-

tween the F and G readings of each. The plus values are those in which F exceeds G and the minus values those in which G exceeds F. The letters preceding the specimen numbers indicate the original channels from which the individual specimens were cut. The two-digit number represents the length of the specimen to the nearest inch and the last number, the serial number of the specimen in the given length.

Geometric Properties

Since the waviness of the flanges would have made any attempt to obtain separate values of the geometric properties of the cross section for each specimen of doubtful value and the median values of the various measurements differed so little from the nominal values, it was decided to use the geometric properties of the nominal cross section in all computations. The amount of error that could result from this practice is indicated by the values of table V, in which are listed the area, the moments of inertia, the radii of gyration, and the distance from the centroid to the center of the back for the nominal and what are termed the "Median," "Small," and "Large" sections. In the computation of these quantities the cross section was assumed to be made up of three rectangles, one back and two flanges. For all four sections the back was assumed to have the width G and the thickness E, using the nominal, median, minimum, or maximum value of the quantity depending on the section in question. For all four sections the width of flange was taken as $(H + I)/2 - E$. The thickness of the flanges was taken as $(A + B + C + D)/4$ for the nominal and median sections, as $(B + C)/2$ for the small section, and as $(A + D)/2$ for the large section. The distance from the axis of symmetry to the midline of a flange was taken as $(F + G)/4 - (A + B + C + D)/8$ for the nominal and the median sections, as $(F + G - A - D)/4$ for the small section and as $(F + G - B - C)/4$ for the large section. Since neither the median, the largest, nor the smallest values of all the various measurements were ever found at the same section, the computed values do not represent conditions at any specific section and certainly do not represent average conditions for any entire specimen. The median section values are good averages but actually no better than the nominal ones. The values for the small and the large sections indicate the extremes of variation possible but they deviate from the nominal more than the actual values at any one section possibly could.

Quality of Material

The quality of the material was determined from tension tests of coupons cut from apparently uninjured portions of the specimens after the column tests. These tests were made in accordance with A.S.T.M. Specification E 8-36 except that the elongation in 2 inches instead of the elongation in 8 inches was measured. Three coupons were obtained from each of the six original channels. The value of Young's modulus, tension yield point, ultimate tensile strength, and percentage elongation in 2 inches are listed in table VII. In a number of the tests, the last-mentioned quantity could not be measured as the specimen broke too close to the end of the gage length.

Since the tests were made on material that had already been subjected to column tests, it might be thought that the results were affected by work hardening. Inasmuch as the axial stresses imposed in the column tests were roughly inversely proportional to the lengths of the columns, any effect of work hardening would be expected to be a function of the length of the specimen from which the tension test coupon was cut. Study of the results of table III will show that no systematic variation of this character is observable. It is therefore believed that the results of the tension tests were unaffected by work hardening.

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TABLE I

Column Test Results

Specimen	Maximum load (lb.)	Twist-axis location: Distance (in.)	Twist-axis location: Precision	Direction of rotation (a)	Deflection parallel to plane of symmetry (in.)	Maximum change in M readings (in.)	Type of failure (b)
90-1	2,250	2.30	fair	+	0.023	0.008	T
90-2	2,360		worthless	-	-.005	.006	T
90-3	2,160	2.75	very good	-	-.045	.016	T
70-1	2,930	2.30	good	+	.036	.008	T
70-2	2,880	2.35	very good	-	-.022	.007	T
70-3	2,960	2.30	fair	-	.025	.006	T
60-1	3,420	2.20	good	-	.037	.009	T
60-2	3,430	2.10	do.	+	.023	.010	T
60-3	3,380	2.15	very good	+	.014	.006	T
50-1	4,240	2.00	excellent	-	-.003	.006	T
50-2	4,380	2.05	very good	-	.014	.005	T
50-3	4,300	2.00	do.	+	.007	.006	T
40-1	5,790	2.00	do.	-	.022	.009	T
40-2	5,800	2.00	good	-	.026	.005	T
40-3	5,800	2.00	do.	-	.010	.016	T
30-1	8,550	1.90	excellent	+	.016	.029	T
30-2	8,730	1.90	do.	+	.018	.094	T
30-3	8,350	1.90	very good	-	.011	.183	T
24-1	12,480	1.70	fair	+	.012	.086	B
24-2	12,450	2.40	poor	-	-.010	.027	B
24-3	11,550	1.90	fair	-	.004	.026	B
22-1	12,120	1.90	good	+	.002	.066	B
22-2	12,750	2.10	poor	-	.006	.095	B
22-3	12,750	1.75	very good	+	-.003	.087	B
20-1	12,850	1.75	do.	-	-.005	.063	B
20-2	13,500	1.65	fair	+	.002	.039	B
20-3	13,050	1.70	do.	+	-.006	.097	B
16-1	13,900	2.60	do.	-	-.003	.085	B
16-2	12,890		worthless	+	-.007	.110	B
16-3	12,350	1.50	good	+	-.017	.067	B
10-1	14,930	.50	fair	-	-.005	.125	B
10-2	15,000	1.25	good	-	-.004	.120	B
10-3	15,130		worthless	+	-.009	.109	B

(a) The + sign indicates that the near end of the antenna appeared to move to the right of an observer; the - sign indicates motion in the opposite direction.

(b) Torsion, T; buckling, B.

TABLE II

Test Results on Short Columns

Specimen	Load carried after buckling (lb.)	M reading at buckle: under maximum shortening (in.)	M reading at buckle: after unloading (in.)	Load when buckling first noticed (lb.)	Flange that buckled
30-3	-	-	-	-	left
24-1	6,150	2-43/64	2-28/64	11,420	right
24-2	7,470	2-44/64	2-26/64	7,990	left
24-3	5,970	2-42/64	2-28/64	9,730	do.
22-1	5,730	2-44/64	2-28/64	10,520	right
22-2			2-29/64		left
22-3	5,570	2-45/64	2-30/64	11,050	right
20-1	6,100	2-44/64	2-31/64	7,000	left
20-2	5,510	2-49/64	2-34/64	10,000	right
20-3	5,670	2-47/64	2-34/64	9,000	do.
16-1	5,930	2-48/64	2-35/64	10,100	left
16-2	6,850	2-37/64	2-29/64	9,000	right
16-3	7,100	2-35/64	2-23/64	11,000	do.
10-1	7,510	2-44/64	2-31/64	10,080	left
10-2	7,700	2-47/64	2-34/64	6,110	do.
10-3	7,300	2-46/64	2-34/64	8,000	right

TABLE III

Critical Loads and Twist-Axis Locations

1	2	3	4	5	6	7	8	9
Specimen	Observed	Computed critical loads				Observed	Computed values	
	P_{cr} (lb.)	P_1 (lb.)	P_2 (lb.)	P_3 (lb.)	P_s (lb.)	Q (in.)	Q_1 (in.)	Q_2 (in.)
90-1	2,250	2,075	2,263	2,282	2,320	2.30	2.360	2.457
90-2	2,366				2,420			
90-3	2,160			2,313	2,300	2.75		
70-1	2,940	2,620	2,852	2,870	3,060	2.30	2.090	2.169
70-2	2,900			2,885	3,100	2.35		
70-3	2,960			2,870	3,045	2.30		
60-1	3,420	3,060	3,307	3,348	3,540	2.20	1.984	2.041
60-2	3,430			3,313	3,620	2.10		
60-3	3,380			3,326	3,445	2.15		
50-1	4,240	3,730	3,996	4,007	4,370	2.00	1.898	1.935
50-2	4,380			4,031	4,450	2.05		
50-3	4,300			4,007	4,325	2.00		
40-1	5,790	4,905	5,190	5,293	5,820	2.00	1.828	1.851
40-2	5,800			5,293	5,940	2.00		
40-3	5,800			5,293	5,860	2.00		
30-1	8,550	7,420	7,682	7,800	9,080	1.90	1.774	1.787
30-2	8,730			7,800	9,020	1.90		
30-3	8,350			7,800	8,890	1.90		
24-1	12,480	10,605	10,843	10,900	13,200	1.70	1.750	1.756
24-2	12,450			15,026	11,330	2.40		
24-3	11,550			11,141	11,840	1.90		
22-1	12,120	12,275	12,503	12,900	14,190	1.90	1.742	1.749
22-2	12,750			14,348	13,020	2.10		
22-3	12,750			12,500	13,290	1.75		
20-1	12,850	14,485	14,690	14,688	14,000	1.75	1.736	1.742
20-2	13,500			14,906	15,240	1.65		
20-3	13,050			14,735	13,760	1.70		
16-1	13,900	21,615	21,757	37,141	14,250	2.60	1.726	1.729
16-2	12,890				12,780			
16-3	12,350			24,125	12,740	1.50		
10-1	14,930	52,500	52,343	470,842	15,220	.50	1.714	1.715
10-2	15,000			81,518	15,840	1.25		
10-3	15,130				15,360			

TABLE IV

Specimen	Load drop		Buckles noted	
	(lb.)	(percent)	pounds below P_{cr}	percent P_{cr}
24-1	6,330	50.8	1,080	91.3
24-2	4,980	40.0	4,460	64.1
24-3	5,580	48.3	1,820	84.2
22-1	6,390	52.6	1,600	86.8
22-2				
22-3	7,180	56.4	1,700	86.8
20-1	6,750	52.5	5,850	54.5
20-2	7,990	59.2	3,500	74.1
20-3	7,380	56.6	4,050	69.0
16-1	7,970	57.4	3,800	72.4
16-2	6,040	47.0	3,890	69.9
16-3	5,250	42.5	1,350	89.0
10-1	7,420	49.7	4,850	67.5
10-2	7,300	48.7	8,890	40.8
10-3	7,830	51.7	7,050	53.4

TABLE V

Comparative Measured and Calculated Values for

Nominal, Median, Small, and Large Sections

(See fig. 22)

Item	Nominal	Median	Percentage variation from nominal	Small section	Percentage variation from nominal	Large section	Percentage variation from nominal
A'	0.100	0.102	2.0	0.101	1.0	0.105	5.0
B	.100	.099	-1.0	.097	-3.0	.101	1.0
C	.100	.098	-2.0	.097	-3.0	.098	-2.0
D	.100	.101	1.0	.100	0	.103	3.0
E	.100	.101	1.0	.099	1.0	.103	3.0
F	2.100	2.110	.48	2.067	-1.57	2.147	2.24
G	2.100	2.104	.19	2.098	-.10	2.112	.57
H	2.050	2.049	-.05	2.042	-.39	2.054	.20
I	2.050	2.048	-.10	2.041	-.44	2.055	.24
area	.6000	.6020	.33	.5845	-2.58	.6234	3.90
I_{yy}	.2672	.2675	.11	.2582	-3.37	.2785	4.23
I_{xx}	.4675	.4709	.73	.4465	-4.50	.4994	6.83
$\bar{x}$	.666	.663	-.45	.658	-1.20	.669	.45
ρ_{yy}	.667	.667	0	.665	-.30	.668	.15
ρ_{xx}	.883	.879	-.45	.839	-4.99	.895	1.36
I_p	.7347	.7384	.50	.7047	-4.09	.7779	5.88

TABLE VI

Variations between Measurements F and G

(All quantities in thousandths of an inch)

Specimen	Max +	Max -	Min +	Min -	Spread
A 20-1	11	--	4	--	7
B 16-3	--	44	--	33	11
D 60-1	6	6	--	--	12
F 10-2	9	4	6	--	13
E 22-1	1	13	--	--	14
C 24-2	25	--	10	--	15
D 60-2	9	9	--	--	18
C 60-3	26	--	7	--	19
E 24-1	5	15	--	--	20
F 20-2	9	12	--	--	21
B 70-1	18	4	--	--	22
C 24-2	30	--	8	--	22
F 22-2	4	18	--	--	22
E 50-1	4	19	--	--	23
C 40-3	--	24	--	1	23
F 30-2	24	--	0	0	24
D 16-1	24	--	0	0	24
F 30-1	17	9	--	--	26
B 70-3	--	32	--	5	27
C 40-2	13	14	--	--	27
F 20-3	21	6	--	--	27
E 50-2	34	--	6	--	28
C 40-1	20	9	--	--	29
F 30-3	15	14	--	--	29
D 90-1	36	--	6	--	30
A 22-1	7	23	--	--	30
F 10-1	21	10	--	--	31
F 10-3	28	5	--	--	33
B 70-2	18	19	--	--	37
F 16-2	32	5	--	--	37
A 90-2	7	30	--	--	37
A 90-3	17	21	--	--	38
E 50-3	39	13	--	--	52

TABLE VII

Physical Properties of Material

Specimen	Young's modulus (lb./sq.in.)	Tensile yield point (lb./sq.in.)	Ultimate tensile strength (lb./sq.in.)	Elongation in 2 inches (percent)
A 90-2	10,330,000	48,000	66,700	15.0
A 90-3	9,840,000	49,000	60,800	--
A 90-3	10,220,000	51,100	62,800	11.5
A 90-3	[a]10,550,000	--	--	--
B 70-1	9,900,000	46,000	61,800	14.5
B 70-2	10,230,000	46,900	62,300	13.5
B 70-3	10,010,000	48,000	60,700	--
C 60-3	11,120,000	48,700	63,100	--
C 60-3	[a]10,650,000	--	--	--
C 40-1	11,070,000	47,800	60,400	--
C 40-2	10,100,000	48,700	58,400	--
D 90-1	10,350,000	49,300	62,500	--
D 60-2	10,000,000	49,100	60,500	11.5
D 60-2a	9,870,000	49,300	63,500	--
E 50-1	10,000,000	47,500	61,000	16.5
E 50-3	9,660,000	48,800	60,500	--
E 24-1	10,220,000	47,500	60,600	15.5
F 30-1	10,530,000	48,000	65,600	--
F 20-2	10,300,000	49,000	66,600	--
F 20-3	9,840,000	48,400	64,000	--
Average	10,200,000	48,400	62,300	14.0
Maximum	11,120,000	51,100	66,700	16.5
Minimum	9,660,000	46,000	58,400	11.5
Deviations percent of mean	9.2 -5.3	11.2 -5.0	7.1 -6.3	17.8 -17.8

[a]Values obtained by National Bureau of Standards on samples supplied by the author after completion of present paper.

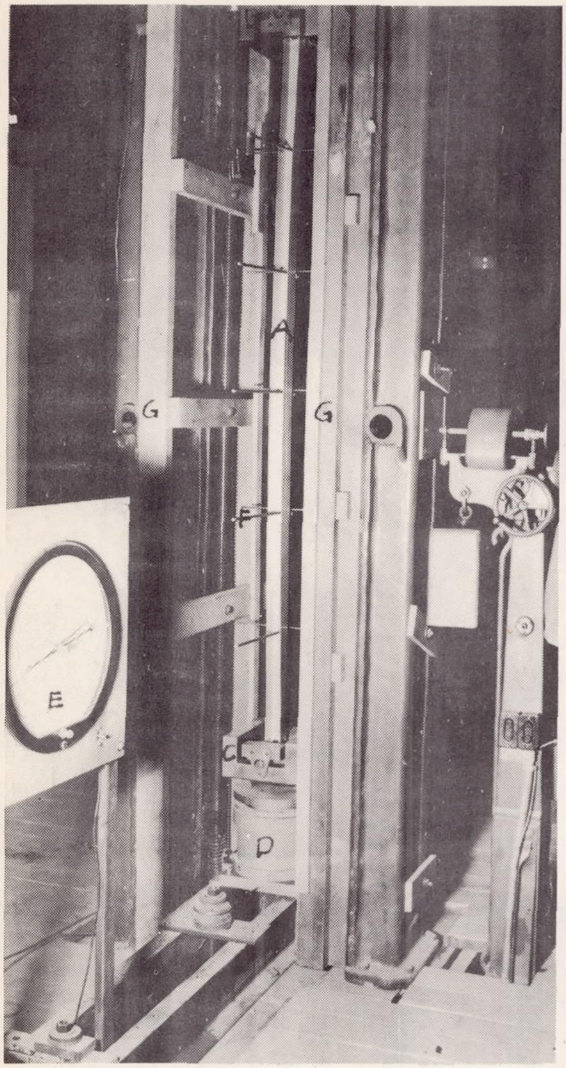


Figure 1.- General arrangement for column tests. Specimen 90-3 under maximum load.

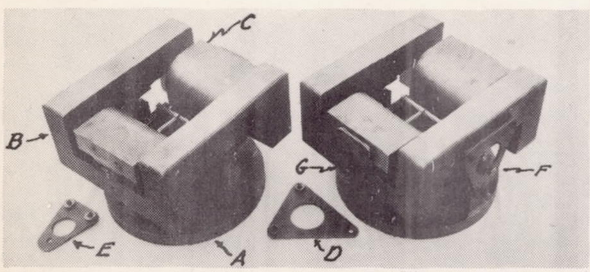


Figure 2.- End-fitting assemblies.

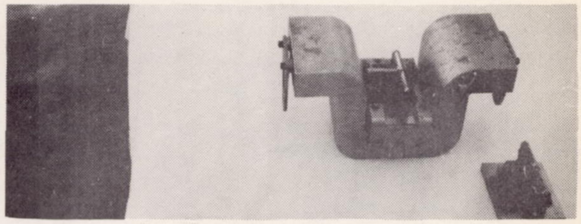


Figure 3.- Saddle.

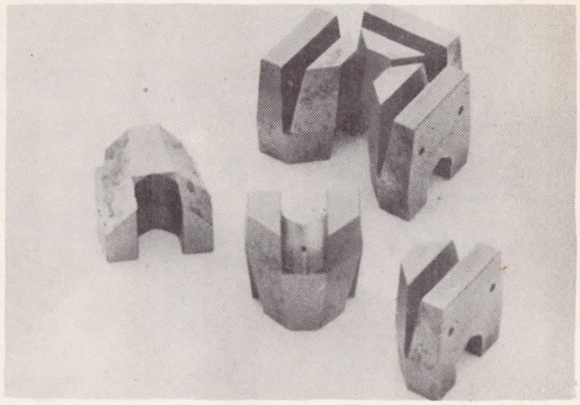


Figure 4.- Bearing blocks.

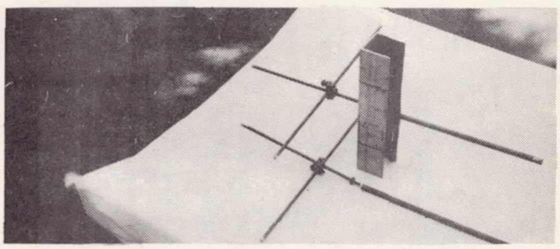


Figure 5.- Antennae.

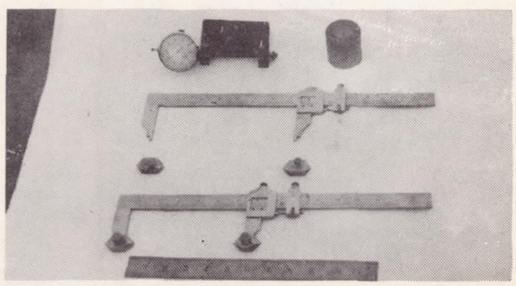


Figure 6.- Calipers.

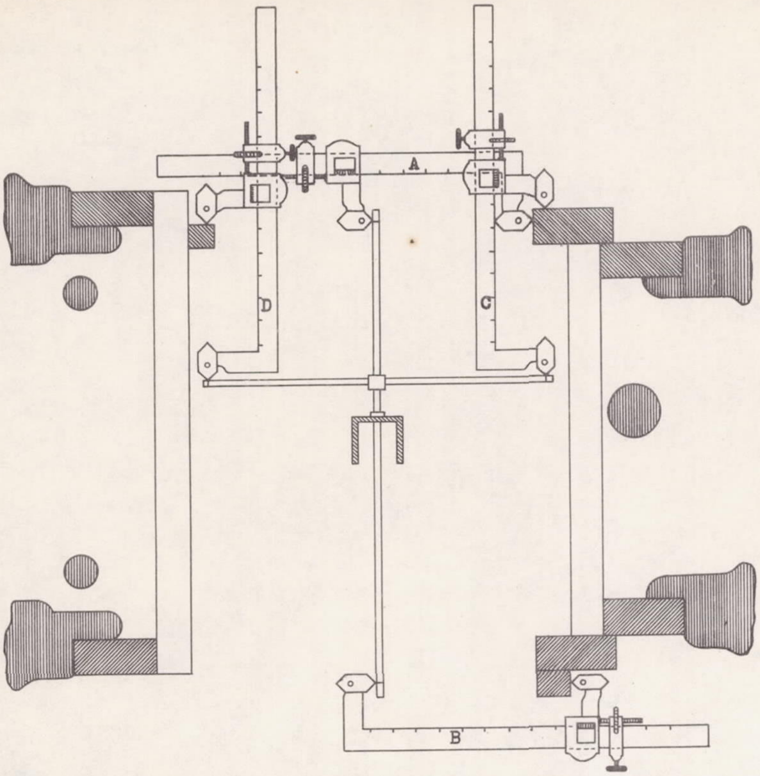


Figure 7.- Caliper positions for vernier readings.

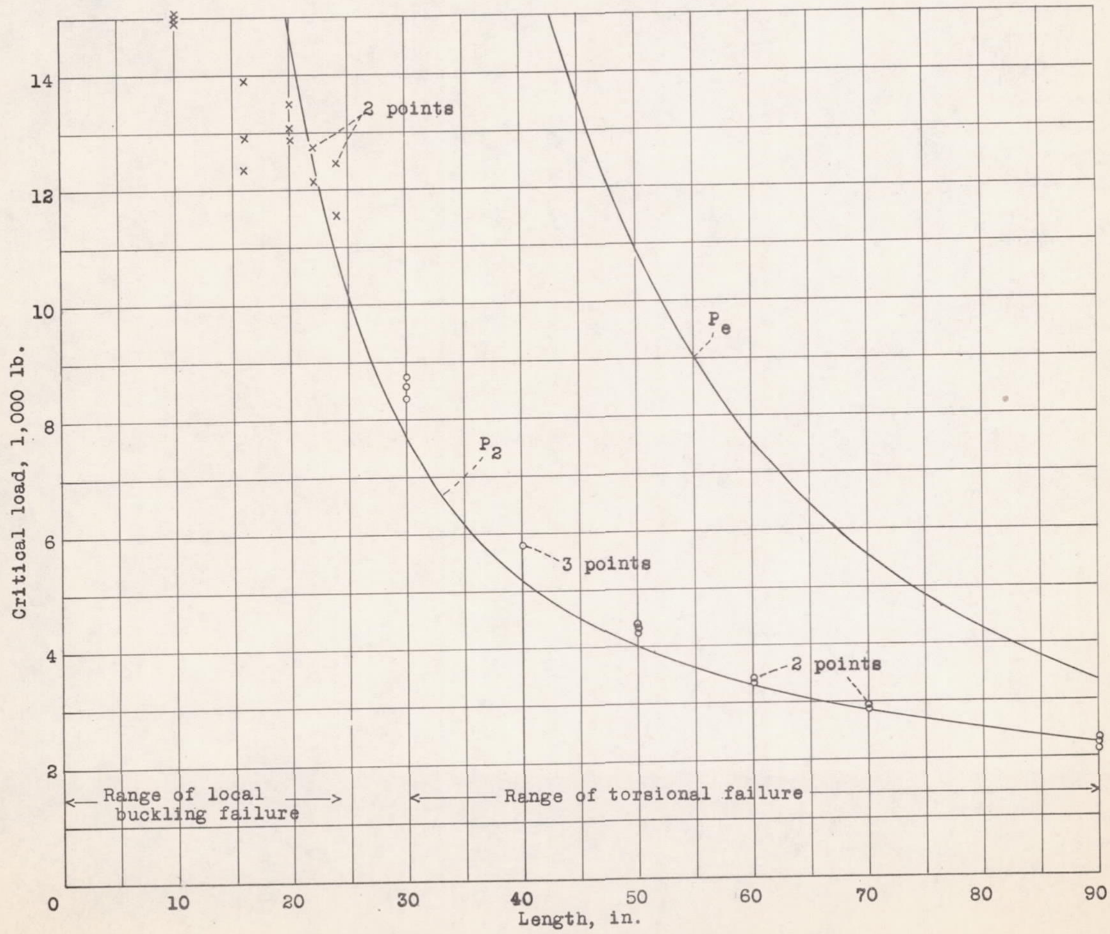


Figure 9.- Critical loads against length.

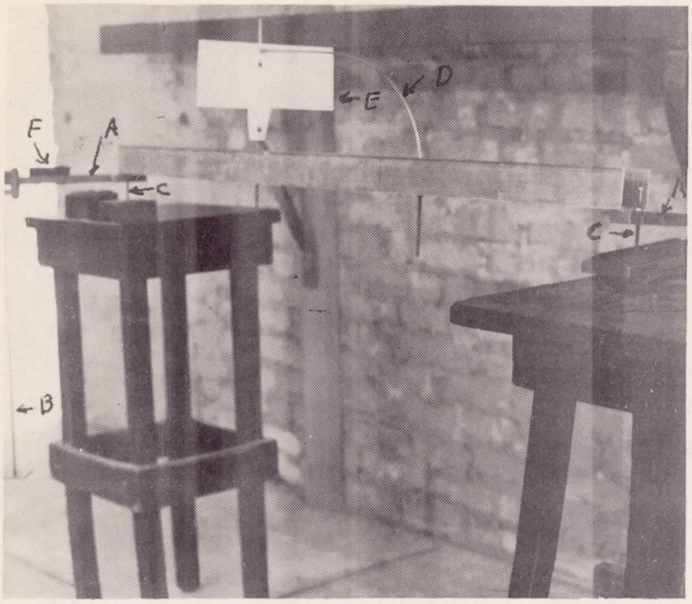


Figure 8.- Torsion-test equipment.

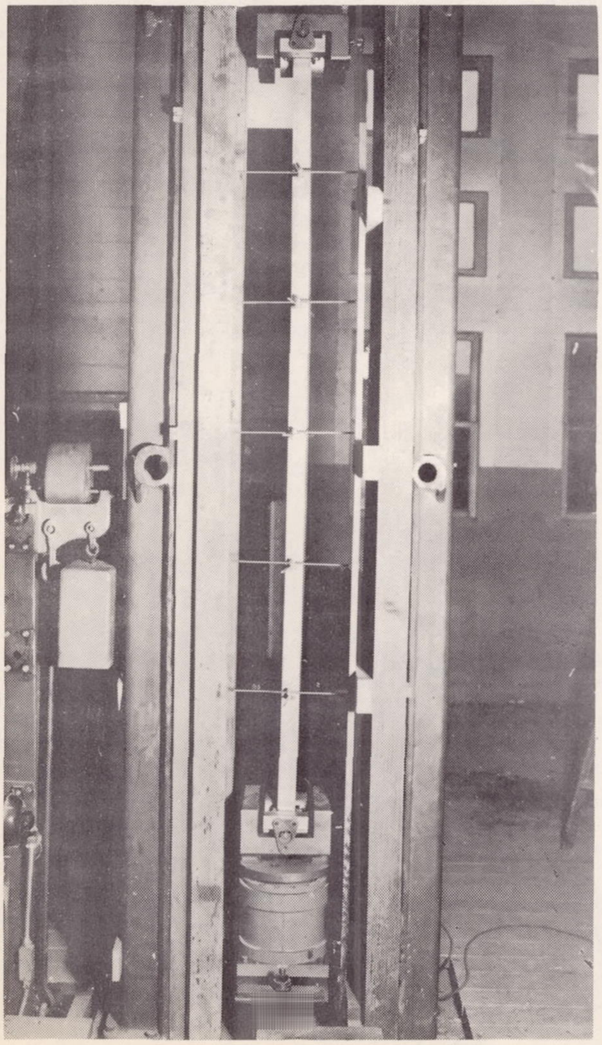


Figure 14.- Specimen 90-3 under maximum load.

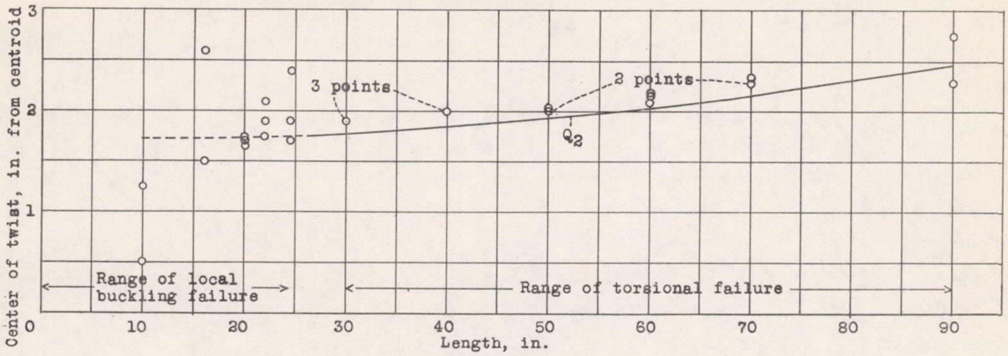


Figure 10.- Location of twist axis against length.

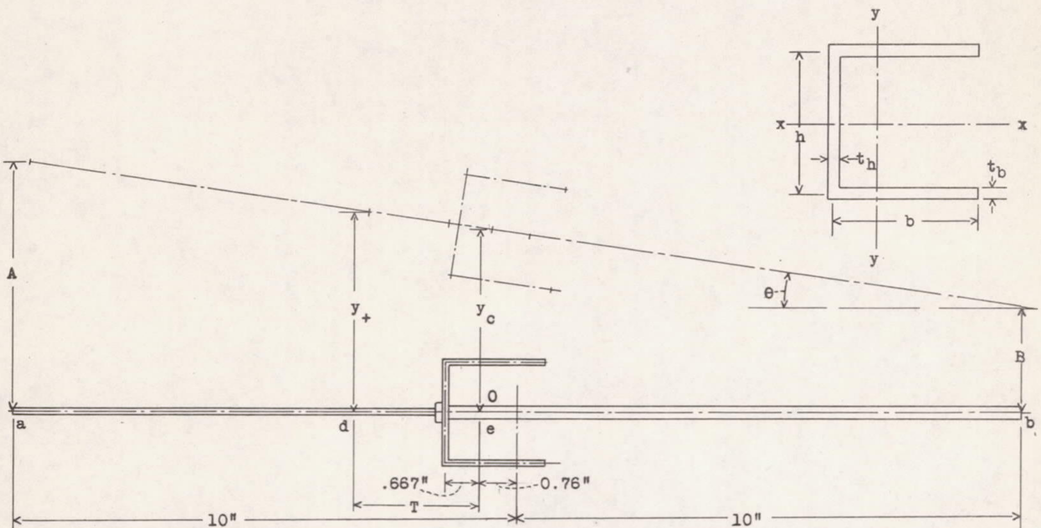


Figure 11.- Geometrical relations between vernier readings and movements of channel cross section.

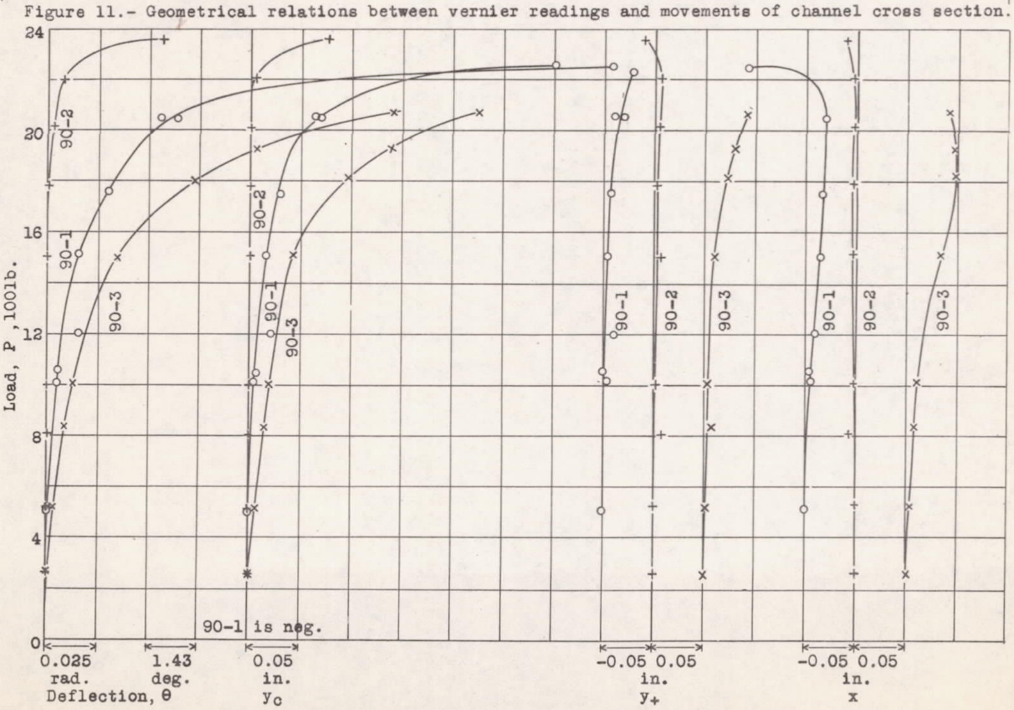


Figure 13.- Deformation curves for 90 inch specimens. Level III.

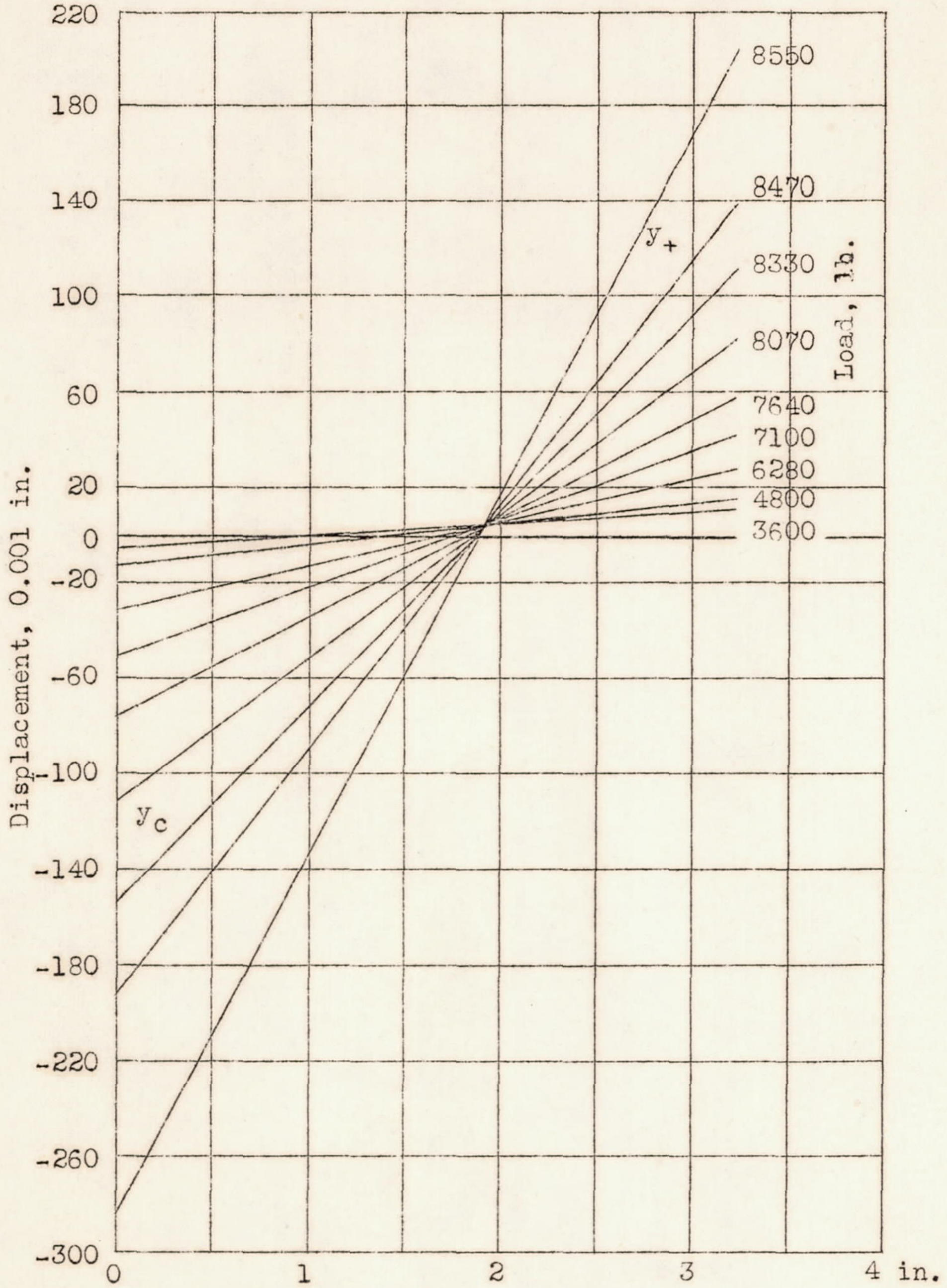
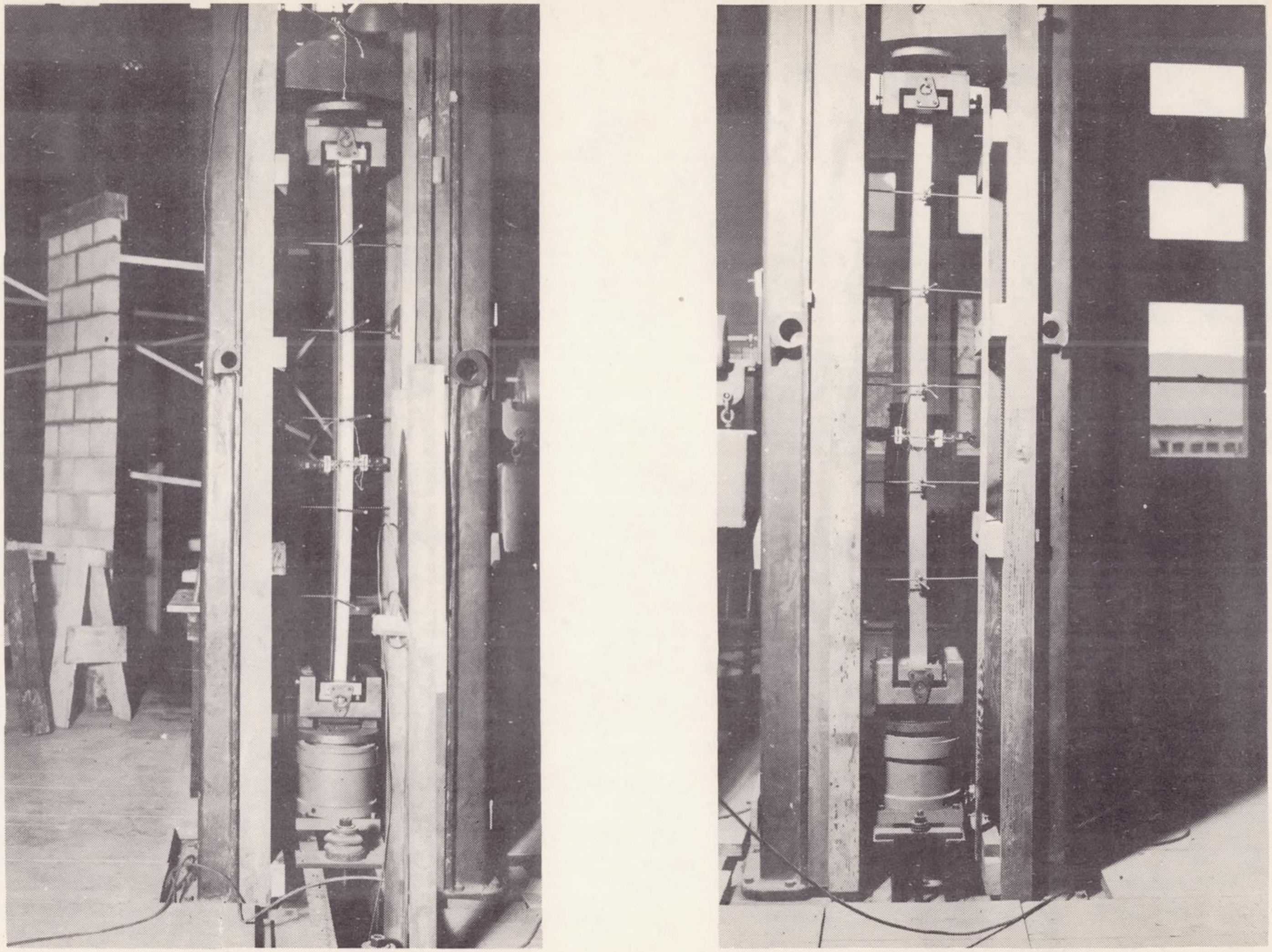


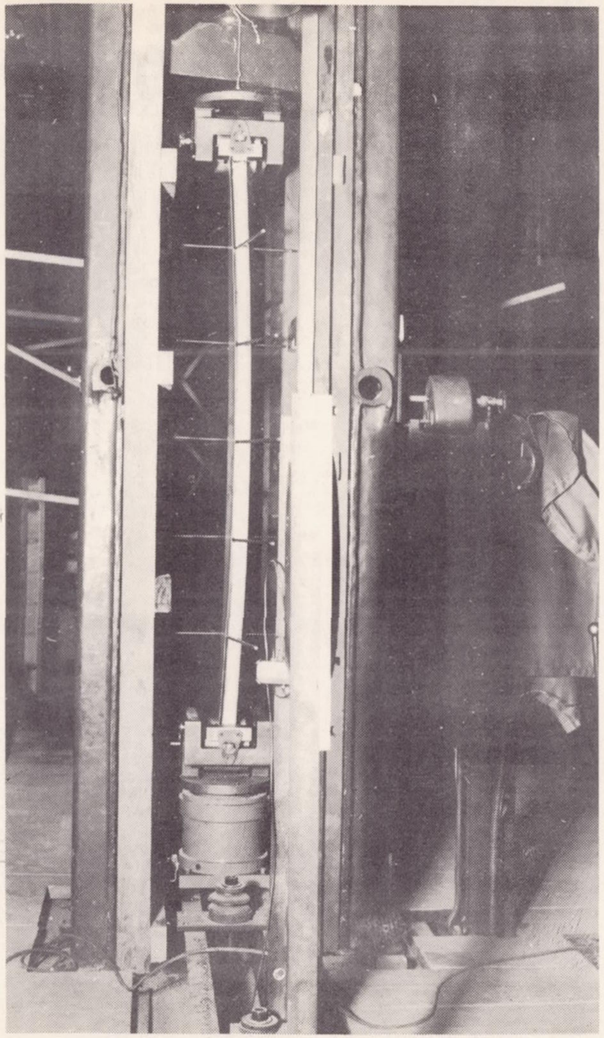
Figure 12.- Vector sheaf. Test 30-1, level III.



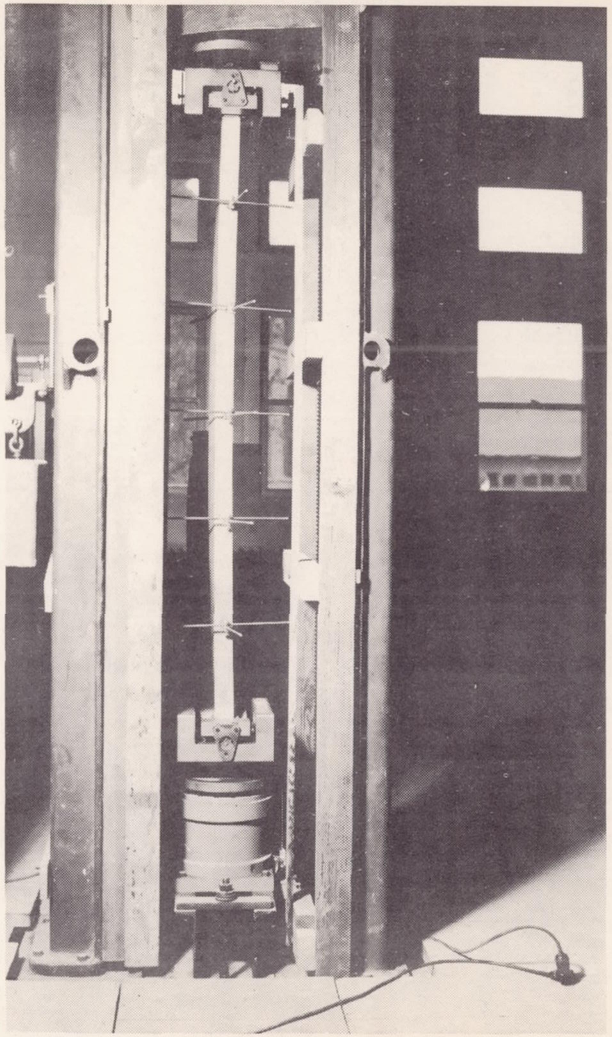
(a) Front view.

(b) Rear view.

Figure 15.- Specimen 70-1 under 2940-pound load before tensiometers were removed.

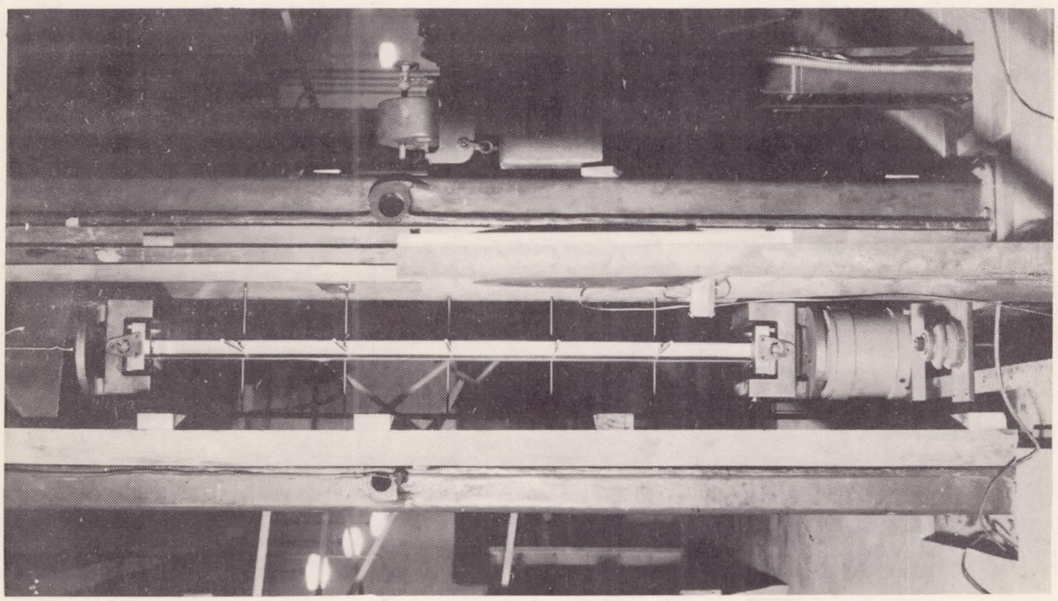


(a) Front view

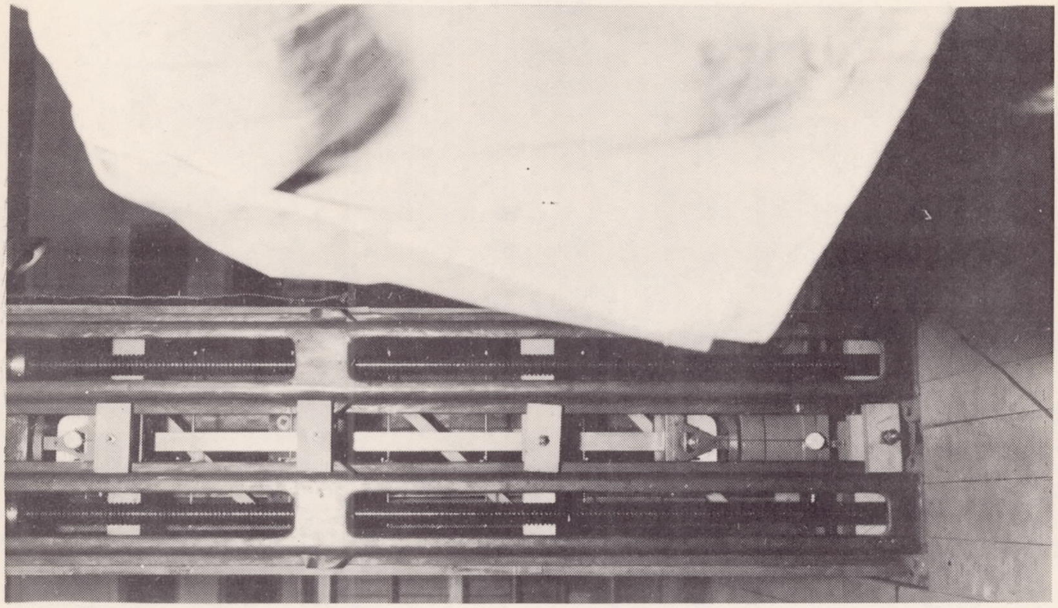


(b) Rear view

Figure 16.- Specimen 70-1 under 2940-pound load after removing tensiometers.

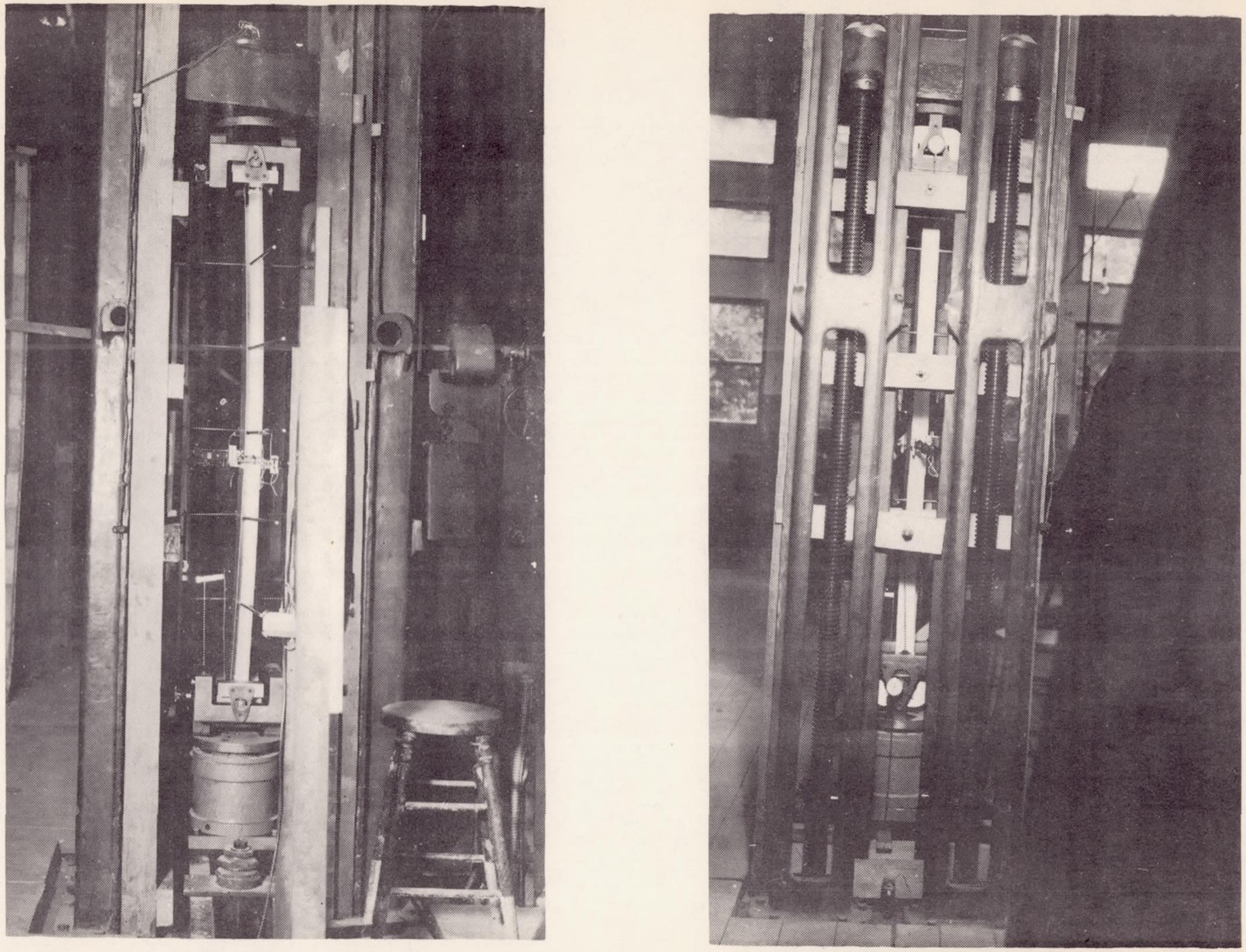


(a) Front view



(b) Side view

Figure 17.- Specimen 70-1 after unloading.



(a) Front view

(b) Side view

Figure 18.- The 50-inch specimen under critical load.

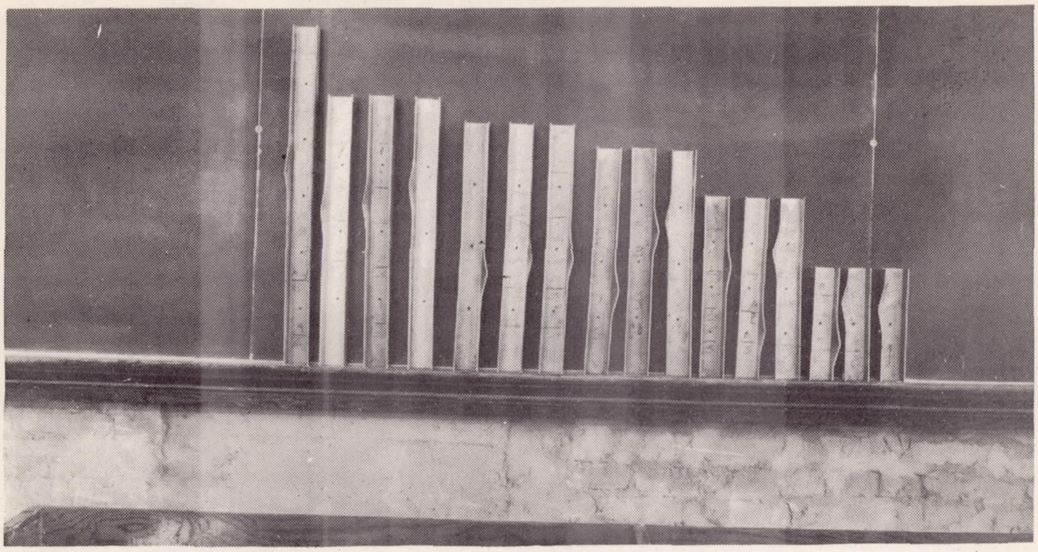


Figure 19.- Short specimens after tests.

Figure 20.- Specimen 10-3 at failure.

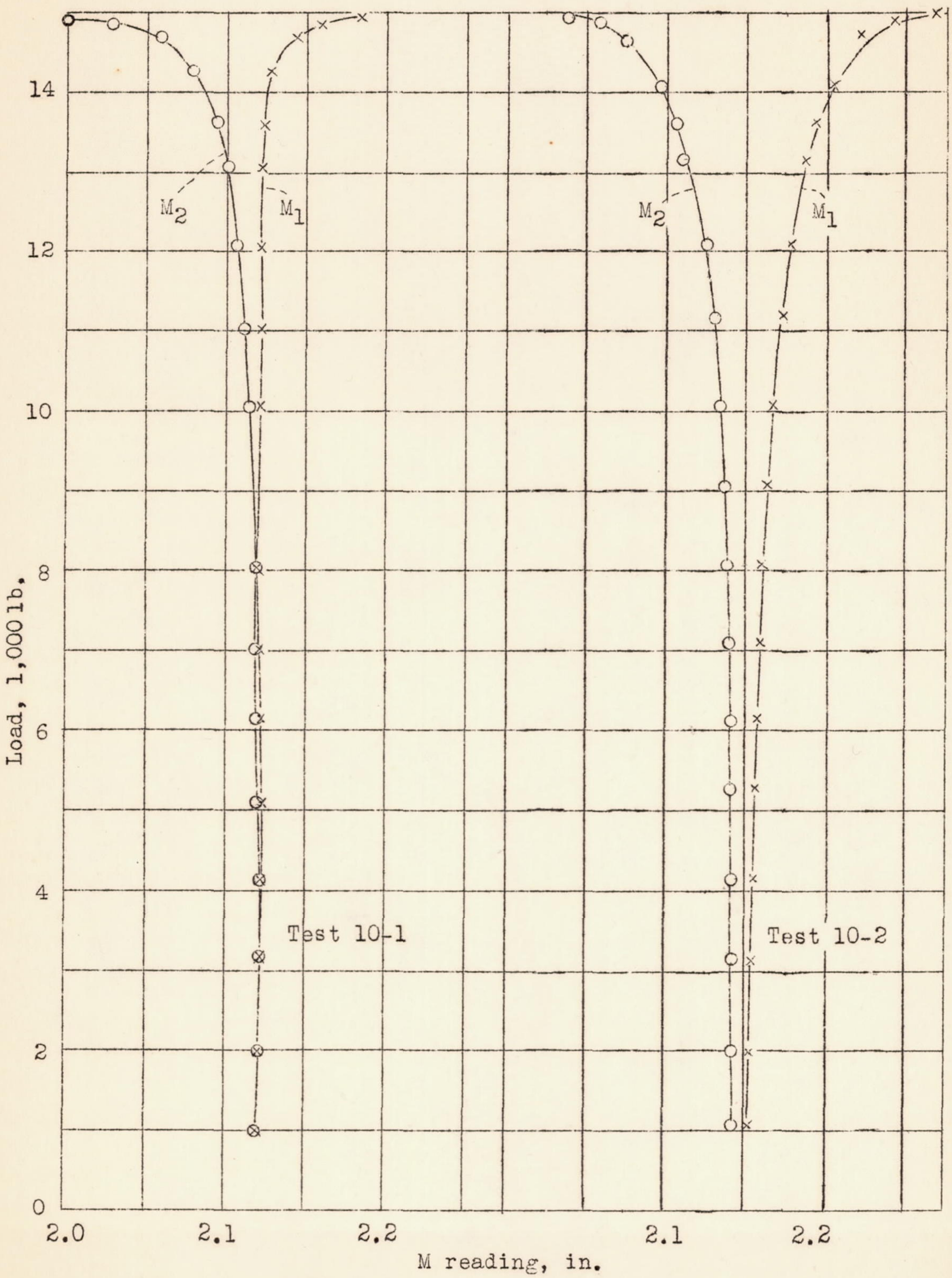


Figure 21.- Variation of M reading with load. Tests 10-1 and 10-2.

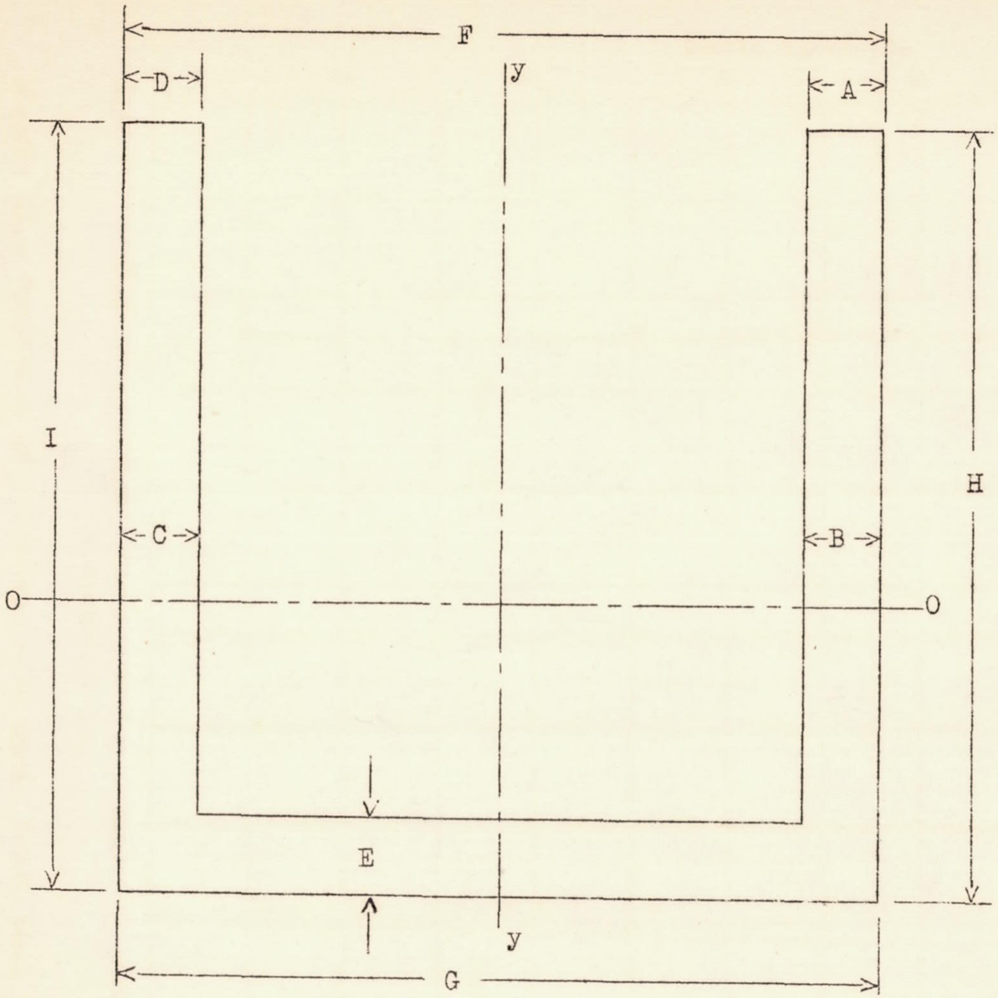


Figure 22.- Locations of check measurements of cross section.

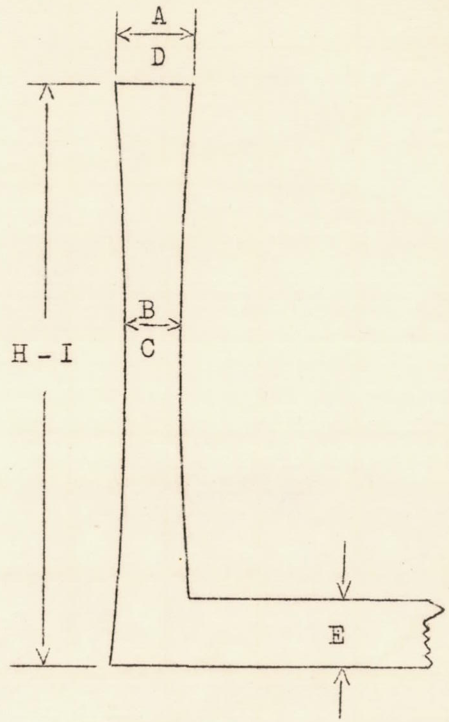


Figure 23.- Section of flange exaggerated.